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On-line Supplement for “Constraint Aggregation in Column Generation Models for Resource-Constrained Covering Problems”

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1. On the update from P_k to P_{k+1}

While the exact incremental calculations used to update P_k to P_{k+1} have a limited theoretical impact on our study, they are essential for the practical speed of 2IDA and they deserve further analysis. We here provide the pseudo-code of the routine from Sec. 4.2. Recall that it starts building \mathcal{P}_{k+1} by lifting \mathcal{P}_k constraints.

Observe that Step (1) of Algorithm 1 generates new configurations \mathbf{a}' by reusing coefficients (c_a^j, N_a^j) from non-split groups $j \neq j^*$ of already-generated configurations $\mathbf{a} \in \mathcal{A}_k$. For any such \mathbf{a} , the lifted configuration \mathbf{a}' is computed by determining the coefficients of the new sub-groups j_1 and j_2 . This is carried out using a dynamic program significantly faster than the original one from Section 3.3.2. Besides only using two decision levels (j_1 and j_2), it also uses a residual capacity reduced from C^+ to $C^+ - \sum_{j \in \{1, \dots, k\} - \{j^*\}} c_a^j$ (it removes the amount consumed by lifted values from $\mathbf{a} \in \mathcal{A}_k$).

Algorithm 1: A Two-Step CG for $\text{DCvr}(\mathcal{P}_{k+1})$: optimize over $\mathcal{P}'_{k+1} \supset \mathcal{P}_k^y$, then over \mathcal{P}_{k+1}^y

Data: Optimal solution $(\alpha^1, \dots, \alpha^k, \beta^1, \dots, \beta^k)$ of \mathcal{P}_k equivalent to $\mathbf{y}_k^* \in \mathcal{P}_k^y$
Result: $\text{lb}_{k+1} = \text{OPT}(\text{DCvr}(\mathcal{P}_{k+1}))$

Lift aggregated solution $(\alpha^1, \dots, \alpha^k, \beta^1, \dots, \beta^k) \in \mathcal{P}_k$ to the space of \mathcal{P}_{k+1} ; – $\alpha^{j_1}, \alpha^{j_2} \leftarrow \alpha^{j^*}$ and $\beta^{j_1}, \beta^{j_2} \leftarrow \beta^{j^*}$ // break j^* into two groups j_1, j_2 ; – keep unchanged the values α^j and β^j for all $j \neq j^*$; repeat for $\mathbf{a} \in \mathcal{A}_k : \mathbf{a}^\top \mathbf{y}_k^* = \mu_a$ do – given current $[\alpha, \beta]$, solve the aggregated multiple-choice pricing (Sec. 3.2.2) with 2 levels (j_1, j_2) and capacities $C^- - \sum_{j \neq j^*} c_a^j$ and $C^+ - \sum_{j \neq j^*} c_a^j$ to lift \mathbf{a} to $\mathbf{a}' \in \mathcal{A}_{k+1}$; – $\mathcal{A}'_{k+1} \leftarrow \mathcal{A}'_{k+1} \cup \{\mathbf{a}'\}$; – optimize over current \mathcal{P}'_{k+1} described by configurations \mathcal{A}'_{k+1} only – update $\text{OPT}(\text{DCvr}(\mathcal{P}'_{k+1}))$ and the current dual solution $[\alpha, \beta]$ until no configuration \mathbf{a}' of negative reduced cost can be found; if $\text{OPT}(\text{DCvr}(\mathcal{P}'_{k+1})) = \text{lb}_k$ return lb_k ; repeat – given current $[\alpha, \beta]$, solve the aggregated multiple-choice pricing (Sec. 3.2.2) on $k+1$ levels and generate a new configuration \mathbf{a} ; – $\mathcal{A}_{k+1} \leftarrow \mathcal{A}_{k+1} \cup \{\mathbf{a}\}$; – optimize current \mathcal{P}_{k+1} described by above \mathcal{A}_{k+1} and update $[\alpha, \beta]$; until no configuration \mathbf{a} of negative reduced cost can be found; return $\text{OPT}(\text{DCvr}(\mathcal{P}_{k+1}))$; <hr/>	}	Step 1: lift \mathcal{P}_k to $\mathcal{P}'_{k+1} \supset \mathcal{P}_{k+1}$
<hr/>	}	Step 2: standard \mathcal{P}_{k+1} optim

1.1. Customizing the Group Split Methods

While the group split operator has no great impact in theory, in practice it is important to take the best split decisions. In Sec. 4.1, we briefly presented:

1. A dichotomic split operator (Sec. 4.1.1) that only maintains the regularity of the generated groups.
2. A splitting strategy (Sec. 4.1.2) that aims at making the current polytope cover a *reference solution* of better quality than the current lower bound.

We here develop this latter strategy in greater detail, using as *reference solution* the 2IDA upper bound in App. 1.1.1 and resp. a dual solution constructed from dual-feasible functions (DFFs) in Sec. 1.1.2 (for CSP and LowWaste-CSP only). These DFFs are well-acknowledged for their speed in pure CSP, *i.e.*, they only require applying a (often piecewise linear) function on the item weights.

1.1.1. Guiding the Split Operator using Upper Bound Solutions We assume that the elements within each group are sorted by increasing weight. The goal is to determine: (i) a group $j^* \in \{1, \dots, k\}$ to split and (ii) a split point i^* such that the first i^* elements of group j^* are assigned to the first (sub-)group and the remaining $n_{j^*} - i^*$ elements to the second

(sub-)group. These decisions rely on a comparison of the current optimal solution \mathbf{y}_k^* of \mathcal{P}_k^y to an outside reference solution. This reference is given by an upper bound solution \mathbf{y}^u (see Sec. 4.3) that indicates a direction of evolution that \mathbf{y}_k^* can follow to reach a better \mathbf{y}_{k+1}^* .

The main idea is that that $\mathbf{y}_k^* \rightarrow \mathbf{y}^u$ is an improving open direction (see Def. 4.1), *i.e.*, there is no legitimate \mathcal{P} constraint that can block a (sufficiently small) advance from \mathbf{y}_k^* to \mathbf{y}^u . We need to identify and break aggregation restrictions that do block such advances. This can be done by evaluating the difference between \mathbf{y}_k^* and \mathbf{y}^u over segments $[i_1, i_2] = \{i_1, i_1 + 1, \dots, i_2\}$ of each group I^j (we only use groups with continuous consecutive indices). More exactly, we define the operator $\Delta_j(i_1, i_2) = \sum_{i=i_1}^{i_2} b_i \cdot (\mathbf{y}^u - \mathbf{y}_k^*)_i$.

Alg. 2 gives the pseudo-code of this split operator. We describe it on the case $\Delta_j(1, n_j) > 0$, *i.e.*, the advance $\mathbf{y}_k^* \rightarrow \mathbf{y}^u$ generates a positive trend (“push”) on the elements of group j . The negative case $\Delta_j(1, n_j) \leq 0$ is symmetric and can be reduced to the positive with the simple inversion from Lines 2-4. If $\Delta_j(i_a, i_b)$ were strictly positive for all $i_a, i_b \in \{1, 2, \dots, n_j\}$, *no* linearity constraint could stop the increasing trend $\mathbf{y}_k^* \rightarrow \mathbf{y}^u$ over group j . The goal is then to break groups j such that the trend $\mathbf{y}_k^* \rightarrow \mathbf{y}^u$ is positive over some $[1, i_a]$, negative over some $[i_a, i_b]$ and (possibly) positive over $[i_b, n_j]$.

Technically, Alg. 2 determines $i_a = \min\{i \in \{1, \dots, n_j\} : \Delta_j(1, i-1) \geq 0, \Delta_j(i, i) < 0\}$ and $i_b = \max\{i \in \{i_a, \dots, n_j\} : \Delta_j(i_a, i) < 0\}$. The split point $i^*(j)$ is either $i_a - 1$ (to cut group j into intervals $[1, i_a - 1]$ and $[i_a, n_j]$) or i_b . If $\Delta_j(1, i_a - 1) > \Delta_j(i_b + 1, n_j)$, we use $i^*(j) = i_a - 1$ and otherwise we use $i^*(j) = i_b$.

The interest in splitting j at $i^*(j)$ is quantified by a heuristic score $h(j)$ initially defined by $\max(\Delta_j(1, i_a - 1), \Delta_j(i_b + 1, n_j))$. Then, we multiply $h(j)$ by the weight spread $w_{\max}^j - w_{\min}^j$, so as to discourage splitting groups with similar weights (Line 13). We finally multiply by 2 the interest of splitting extremal groups 1 and k (Line 14), because the smallest and the largest items can have a higher importance, *e.g.*, in CSP, many small (resp. large) items will have a dual value of 0 (resp. 1) at optimality and this has a strong influence on the way all other groups are determined.

1.1.2. Guiding the Split Operator using Dual-Feasible Functions We recall that $f : [0, C] \rightarrow [0, 1]$ is a *dual-feasible function* (DFF) if and only if

$$\sum_{i \in I} a_i w_i \leq C \implies \sum_{i \in I} a_i f(w_i) \leq 1$$

Algorithm 2: Group Split Operator Guided by an Upper Bound Reference Solution**Data:** \mathbf{y}_k^* and \mathbf{y}^u **Result:** (i) group j^* to split, (ii) the number i^* of elements of the first sub-group

```

1 for  $j \leftarrow 1$  to  $k$  do
2   if  $\Delta_j(1, n_j) < 0$  then
3      $\mathbf{y}_k^* \leftarrow -\mathbf{y}_k^*$  // simple inversion to reduce the
4      $\mathbf{y}^u \leftarrow -\mathbf{y}^u$  // negative case to a positive case
5      $i_a \leftarrow \min \{i \in \{1, \dots, n_j\} : \Delta_j(1, i-1) \geq 0, \Delta_j(i, i) < 0\}$ 
6      $i_b \leftarrow \max \{i \in \{i_a, \dots, n_j\} : \Delta_j(i_a, i) < 0\}$ 
7     if  $\Delta_j(1, i_a-1) > \Delta_j(i_b+1, n_j)$  then
8        $i^*(j) \leftarrow i_a-1$  // Choose sub-groups  $[1, i_a-1]$  and  $[i_a, n_j]$  for group  $j$ 
9        $h(j) \leftarrow \Delta_j(1, i_a-1)$ 
10    else
11       $i^*(j) \leftarrow i_b$  // Choose sub-groups  $[1, i_b]$  and  $[i_b+1, n_j]$  for group  $j$ 
12       $h(j) \leftarrow \Delta_j(i_b+1, n_j)$ 
13       $h(j) \leftarrow h(j) \cdot (w_{\max}^j - w_{\min}^j)$  // Discourage splitting groups with similar weights
14      if  $j = 1$  or  $j = k$  then  $h(j) \leftarrow h(j) \cdot 2$  // Encourage splitting extremal groups
15      if  $h(j) > h(j^*)$  then  $j^* \leftarrow j$  // Initially,  $j^*$  was 0 and  $h(j^*)$  was  $-\infty$ 
16 return  $(j^*, i^*(j^*))$ 

```

holds for any index set I , any $a_i \in \mathbb{Z}^+$ and $w_i > 0$ ($\forall i \in I$). This ensures that a dual solution $y_i = f(w_i)$ is feasible in CSP or LowWaste-CSP, but not in MultLen-CSP (because the patterns do not have all the same cost 1).

All classical DFFs surveyed in [3] have a piece-wise linear form. A well-known DFF is the identity function $f(x) = \frac{x}{C}$ which produces the dual feasible solution $y_i = \frac{w_i}{C}$. However, most DFFs are given by staircase functions. To guide the split operator using high-quality reference solutions, we always choose from [3] the DFF f with 10 intervals (pieces) that yields the highest objective value.

The proposed DFF-based split operator identifies $k_D \leq 10$ intervals of $[0, C]$ before starting the construction of \mathcal{P}_k , i.e., these k_D intervals are chosen so that f is linear over each of them. When 2IDA reaches iteration $k = k_D$, the polytope \mathcal{P}_k^y has to include the solution $y_i = f(w_i)$, because f is linear over all groups of \mathcal{P}_k^y . As such, the 2IDA bound $\text{lb}_k = \text{OPT}(\text{DCvr}(\mathcal{P}_k))$ dominates the DFF bound $\sum_{i \in I} b_i f(w_i)$ associated to f .

Table 2: More Detailed Results

Cutting Stock instance by instance results

In brackets, we provide the total (lower and, respectively, upper bounding) time in milliseconds. The key word CG indicates that the final Column Generation phase has been executed in the end, as the bounds lb and ub were already very close (conditions described in the paper).

Instance name/Id	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	ColGen (ms)								
	lb [T _{lb}]	ub [T _{ub}]	lb [T _{lb}]	ub [T _{ub}]	lb [T _{lb}]	ub [T _{ub}]	lb [T _{lb}]	ub [T _{ub}]	lb [T _{lb}]	ub [T _{ub}]	Opt/Lb								
HARD1	55.388[380]	[380]	56.037[8399]	56.989[27065]							56.039[77960,items=655]								
HARD2	55.558[252]	[252]	55.978[9834]	CG55.999[32683]							55.981[77746,items=665]								
HARD3	54.416[261]	[261]	54.922[5383]	CG54.999[28100]							54.923[79483,items=672]								
HARD4	55.507[251]	[251]	56.408[5866]	CG56.913[34104]							56.430[71173,items=628]								
HARD5	54.808[261]	[261]	55.523[7084]	55.994[27177]							55.523[75361,items=657]								
HARD6	55.445[253]	[253]	56.096[7945]	56.884[26351]							56.096[71733,items=630]								
HARD7	53.992[274]	[274]	54.198[5068]	57.406[31739]	54.198[53138]	CG55 [53630]					54.248[78884,items=743]								
HARD8	55.54 [261]	[261]	56.088[9748]	56.94 [28356]							56.083[72737,items=632]								
HARD9	55.005[262]	[262]	55.473[6948]	55.933[27088]							55.473[78066,items=670]								
HARD0	54.403[265]	[265]	55.001[4545]	55.999[24724]							55.006[80926,items=686]								
m01-1	48.62 [5]	[5]	54 [8]	54 [96]							54 [117,items=111]								
m01-2	52.309[2]	[2]	52.309[5]	77 [24]	55 [25]	55 [49]					55 [88,items=134]								
m01-3	49.39 [1]	[1]	49.39 [4]	54 [199]	54 [200]	54 [200]					54 [81,items=112]								
m01-4	52.643[1]	[1]	56 [2]	56 [69]							56 [75,items=134]								
m01-5	44.72 [1]	[1]	44.72 [3]	CG45.25 [368]							45.25 [206,items=204]								
m01-6	52.041[1]	[1]	56 [2]	60 [35]	58 [36]	58 [37]					58 [89,items=108]								
m01-7	50.459[1]	[1]	50.464[2]	61.5 [40]	50.464[1]	61.5 [41]	53 [42]	53 [60]			53 [84,items=153]								
m01-8	52.969[1]	[1]	54 [2]	56 [58]	54 [59]	56 [59]	54.667[61]	55 [63]			54.714[3] 75,items=181]								
m01-9	49.77 [1]	[1]	49.77 [2]	51.5 [165]	49.77 [167]	51.5 [167]	51.5 [168]	51.5 [168]			51.5 [85,items=175]								
m01-10	54.418[1]	[1]	55 [2]	60 [46]	55 [47]	60 [47]	57.5 [49]	58 [51]			57.5 [61,items=127]								
m20-1	55.347[4]	[4]	55.371[9]	63.435[80]	55.492[82]	62.784[83]	58.667[86]	59 [126]			59 [95,items=148]								
m20-2	57.756[1]	[1]	57.756[3]	80 [16]	58.379[18]	CG61 [43]					61 [61,items=149]								
m20-3	62.092[1]	[1]	64 [3]	72 [62]	68 [64]	68 [67]					68 [38,items=106]								
m20-4	56.978[1]	[1]	56.978[3]	80 [17]	56.978[17]	CG60.5 [40]					60.5 [51,items=128]								
m20-5	58.541[1]	[1]	58.541[4]	79 [15]	58.557[17]	CG62.5 [37]					62.5 [37,items=99]								
m20-6	59.427[1]	[1]	61 [3]	67 [78]	65 [80]	65 [82]					65 [39,items=110]								
m20-7	61.204[1]	[1]	61.204[3]	74.5 [18]	61.797[19]	74.5 [19]	65.5 [21]	CG66.5 [36]			66.5 [32,items=96]								
m20-8	57.327[1]	[1]	57.327[3]	78.5 [16]	60.5 [17]	61 [66]					60.5 [48,items=125]								
m20-9	56.796[1]	[1]	56.796[3]	78 [15]	56.796[16]	78 [16]	60 [17]	CG61 [37]			61 [42,items=113]								
m20-10	60.429[1]	[1]	60.429[3]	82 [14]	60.429[15]	CG64 [38]					64 [48,items=128]								
m35-1	66.827[4]	[4]	74.431[8]	85 [22]	74.431[25]	CG77.5 [39]					77.5 [28,items=55]								
m35-2	66 [2]	[2]	75.214[5]	88 [17]	77 [19]	CG77.5 [52]					77.5 [23,items=54]								
m35-3	67.653[2]	[2]	73 [4]	86 [38]	80.333[40]	CG80.5 [43]					80.5 [21,items=53]								
m35-4	66.194[1]	[1]	75.3 [3]	76 [35]							75.3 [22,items=63]								
m35-5	68.51 [1]	[1]	73.3 [2]	88 [11]	80.5 [12]	81 [17]					80.5 [18,items=55]								
m35-6	65.677[1]	[1]	67.281[3]	92 [8]	77 [10]	77 [34]					77 [20,items=68]								
m35-7	63.296[1]	[1]	70 [3]	70 [73]							70 [20,items=77]								
m35-8	66.051[1]	[1]	80 [2]	84 [38]	82.5 [40]	83 [40]					82.5 [13,items=47]								
m35-9	64.854[1]	[1]	76 [34]	76 [34]							75.5 [23,items=68]								
m35-10	63.592[1]	[1]	66.096[3]	89.5 [10]	66.096[11]	CG71.5 [31]					71.5 [22,items=82]								
vbInd-30p0	89.448[179]	[179]	89.448[687]	89.819[1567]							89.4478 [2384,items=96]								
vbInd-443p20	28.515[3]	[3]	28.515[11]	35.769[2275]	28.515[281]	35.769[2321]	28.521[2332]	33.731[3084]	28.525[3095]	33.697[3209]	28.534[3218]	33.289[3596]	28.535[3603]	32.66 [4126]	28.542[4136]	32.66 [4216]	28.55 [4222]	28.862[4381]	
vbInd-443p21	39.507[32]	[32]	39.507[1020]	68.69[2905]	39.508[492]	CG39.939[6245]													
vbInd-44p1	19 [0]	[0]	19 [0]	19 [25]															
vbInd-5p11	20.536[1]	[1]	20.539[2]	41 [649]	20.625[650]	20.627[757]													
vbInd-6p20	12.501[0]	[0]	12.505[2]	24.667[9]	12.525[11]	CG12.671[19]													
vbInd-7p14	69.461[5]	[5]	69.469[11]	138.62[47]	69.495[56]	CG69.682[75]													
vbInd-7p17	8.7373[96]	[96]	8.7373[766]	8.8857[897]															
vbInd-7p18	90.966[11]	[11]	90.978[27]	150 [75]	90.188[81]	CG90.902[123]													
vbInd-11p4	100.91[75]	[75]	100.91[1560]	100.91[12250]															
vbInd-12p19	22.887[12]	[12]	22.887[60]	26.345[142]	22.888[163]	CG22.929[206]													
vbInd-12p24	57.42 [52]	[52]	57.42 [129]	66.688[327]	57.42 [338]	CG57.612[354]													
vbInd-3b2	55.093[31]	[31]	55.093[74]	55.923[6802]															
vbInd-18p22	32.027[35]	[35]	32.027[256]	32.986[403]															
vbInd-25p0	114.89[122]	[122]	114.89[263]	CG114.91[1263]															
vbInd-28p0	187.73[275]	[275]	187.73[655]	187.98[2043]															
vbInd-416p6	36.912[0]	[0]	36.912[1]	62 [107]	36.93 [108]	56.2 [171]	36.95 [172]	56.2 [182]	37.104[183]	37.7 [236]									
vb20c1p1.dat	294.94[115]	[115]	295.02[225]	714.5 [467]	295.02[225]	714.5 [467]	CG295.85[799]												
vb20c1p2.dat	459.23[6]	[6]	460.62[12]	839 [194]	471 [195]	476 [975]	471 [977]	476 [1011]	471 [1013]	476 [1047]	473.5 [1050]	473.5 [1084]							
vb20c1p3.dat	344.67[105]	[105]	344.76[316]	812 [570]	345.68[582]	CG347.97[743]													
vb20c1p4.dat	377.34[86]	[86]	377.44[190]	800 [419]	377.34[86]	CG394 [729]													
vb20c1p5.dat	353.72[141]	[141]	354.46[306]	739.5 [553]	354.68[676]	CG361.1 [913]													
vb20c2p1.dat	248.61[219]	[219]	248.63[462]	682.5 [798]	248.66[862]	CG249.37[1437]													
vb20c2p2.dat	177.86[377]	[377]	177.86[764]	CG177.96[4884]															
vb20c2p3.dat	377.35[7]	[7]	377.44[17]	916.5 [233]	382.41[238]	597.25[593]	383.1 [595]	421.75[737]	383.18[740]	418.17[846]	383.29[848]	385.75[989]	383.29[1005]	383.71[1111]					
vb20c2p4.dat	261.43[145]	[145]	261.45[383]	647 [700]	261.46[759]	CG261.95[1034]													
vb20c2p5.dat	235.35[224]	[224]	235.35[463]	296.5 [2181]	235.35[2240]	296.5 [2244]	CG235.95[4081]												
vb20c3p1.dat	146.34[270]	[270]	146.34[1316]	146.82[2667]															
vb20c3p2.dat	104.42[429]	[429]	104.42[1056]	104.42[3016]															
vb20c3p3.dat	141.51[367]	[367]	141.51[1387]	141.87[3288]															
vb20c3p4.dat	147.51[385]	[385]	147.51[1064]	147.96[3201]															
vb20c3p5.dat	71.776[532]	[532]	71.776[1387]	71.79 [3935]															
vb20c4p1.dat	228.83[160]	[160]	228.84[414]	354 [1171]	228.84[1242]	CG228.99[1516]													
vb20c4p2.dat	275.33[118]	[118]	275.34[260]	669 [512]	275.4 [542]	CG276.09[841]													
vb20c4p3.dat	254.42[161]	[161]	254.43[340]	686 [612]	254.45[836]	CG254.95[1148]													
vb20c4p4.dat	229.95[130]	[130]	229.96[317]	736.25[579]	229.97[647]	CG230.01[1009]													
vb20c4p5.dat	242.59[121]	[121]	242.59[264]	379 [949]	242.61[1132]	CG243.39[1425]													
vb20c5p1.dat	116.02[328]	[328]	116.02[1384]	116.97[2472]															
vb20c5p2.dat	139.98[281]	[281]	139.98[1673]	139.98[2914]															
vb20c5p3.dat	143.87[255]	[255]	143.87[1849]	143.87[3104]															

Instance by instance results for Multiple Length Cutting Stock with $\mu(0.7C)=0.6$ and $\mu(C)=1$						
Instance name/id	k=1		k=2		ColGen(T_{ms})	
	lb [T_{lb}]	ub [T_{ub}]	lb [T_{lb}]	ub [T_{ub}]	Opt/Lb	
'HARD1'	47.5 [941]	198.6 [5290]	47.7 [70579]	CG51.5 [133995]	51.4253[84540, iters=746]	
'HARD2'	47.7 [1015]	198.6 [5114]	47.8 [69170]	CG51.5 [129665]	51.442[82495, iters=741]	
'HARD3'	46.8 [972]	198.6 [5156]	46.9 [70460]	CG50.6 [134708]	50.5923[89323, iters=788]	
'HARD4'	47.6 [963]	198.6 [5060]	47.8 [68995]	CG52.1 [126929]	52.0829[81661, iters=737]	
'HARD5'	47.1 [1060]	198.6 [5231]	47.3 [79083]	CG51.5 [143326]	51.484[89721, iters=791]	
'HARD6'	47.6 [925]	198.6 [5023]	47.7 [78906]	CG51.9 [140635]	51.862[85698, iters=760]	
'HARD7'	46.5 [1106]	198.6 [5418]	46.6 [89637]	CG49.2 [160297]	49.1857[98024, iters=828]	
m01-1	41.7 [6]	93.6 [15]	41.7 [22]	CG49.3 [153]	49.3[128, iters=139]	
m01-2	45.7 [2]	99.6 [8]	48.5 [11]	CG53 [105]	53[97, iters=166]	
m01-3	42.4 [2]	95.6 [8]	42.4 [12]	CG48.2 [110]	48.2[85, iters=161]	
m01-4	45.1 [2]	95.6 [8]	45.1 [12]	CG53.2 [79]	53.2[64, iters=125]	
m01-5	38.6 [2]	99.6 [7]	40.5 [10]	CG44.3 [149]	44.3[177, iters=212]	
m01-6	45 [2]	99.6 [8]	47.3 [12]	CG51.8 [113]	51.8[94, iters=147]	
m01-7	43.3 [1]	95.6 [5]	43.3 [8]	CG50.5 [68]	50.5[86, iters=167]	
m20-1	47.4 [5]	98.6 [12]	48.4 [20]	CG56.6 [106]	56.6[111, iters=172]	
m20-2	50 [2]	99.6 [7]	53.2 [10]	CG58.7 [76]	58.7[62, iters=158]	
m20-3	53.1 [2]	97.6 [7]	55.2 [11]	CG64.8 [54]	64.8[44, iters=123]	
m20-4	48.4 [2]	97.6 [7]	50.5 [11]	CG58.9 [62]	58.9[57, iters=133]	
m20-5	50.1 [2]	97.6 [7]	50.1 [12]	CG59.5 [62]	59.5[67, iters=161]	
m20-6	50.7 [2]	97.6 [7]	50.7 [11]	CG60.2 [58]	60.2[44, iters=125]	
m20-7	52.3 [2]	97.6 [6]	52.3 [10]	CG63.9 [49]	63.9[46, iters=119]	
m35-1	58.3 [3]	100 [6]	62.6 [10]	CG73.9 [45]	73.9[33, iters=68]	
m35-2	57.9 [2]	100 [6]	60.5 [9]	CG71.5 [53]	71.5[36, iters=96]	
m35-3	58.7 [1]	100 [4]	61.9 [7]	CG73.7 [28]	73.7[28, iters=89]	
m35-4	57.3 [1]	98.8 [6]	60.6 [8]	CG72.7 [37]	72.7[27, iters=83]	
m35-5	58.9 [1]	99 [5]	64.9 [7]	CG75.3 [31]	75.3[19, iters=73]	
m35-6	58 [1]	99.6 [5]	62.9 [7]	CG72.2 [26]	72.2[26, iters=103]	
m35-7	56.6 [1]	100 [4]	60 [7]	CG67.2 [50]	67.2[25, iters=86]	
CSTR30p0	76.7 [240]	703 [527]	76.7 [1773]	CG76.7 [2575]	76.6757[3176, iters=133]	
CSTRd43p20	24.5 [6]	112.8 [2217]	24.5 [2250]	CG24.6 [4332]	24.5333[3035, iters=121]	
CSTRd43p21	33.9 [178]	153.4 [4818]	33.9 [7591]	CG34.2 [10094]	34.1531[6717, iters=142]	
CSTRd4p1	19 [0]	19 [55]			19[29, iters=5]	
CSTR5p11	17.7 [3]	39.4 [1972]	17.8 [1983]	CG18 [3008]	17.9062[1126, iters=15]	
CSTR6p20	10.8 [3]	42.6 [29]	10.8 [42]	CG10.8 [62]	10.7818[20, iters=18]	
CSTR7p14	59.7 [7]	250.4 [39]	60.2 [66]	CG60.9 [87]	60.885[82, iters=16]	
CSTR20b50c1p1.dat	252.9 [234]	938.6 [422]	254.9 [1649]	CG265.6 [1807]	265.55[1293, iters=36]	
CSTR20b50c1p2.dat	397.7 [9]	933.8 [143]	398.3 [178]	CG455.5 [266]	455.5[706, iters=30]	
CSTR20b50c1p3.dat	297.3 [284]	914 [499]	299.4 [2123]	CG314.7 [2418]	314.625[1140, iters=37]	
CSTR20b50c1p4.dat	323.5 [193]	967.8 [375]	324.9 [1360]	CG337.2 [1698]	337.175[1173, iters=37]	
CSTR20b50c1p5.dat	302.6 [359]	1000.2[564]	306.3 [2073]	CG328.6 [2323]	328.6[1429, iters=36]	
CSTR20b50c2p1.dat	213.4 [472]	910.3 [799]	214.7 [2803]	CG219.3 [3300]	219.3[2633, iters=57]	
CSTR20b50c2p2.dat	152.5 [609]	847.4 [1038]	152.5 [4204]	CG152.6 [5767]	152.578[4885, iters=74]	
CSTR50b50c1p01.dat	802.7 [501]	2453.8[1091]	803.4 [3148]	CG866.3 [4222]	866.3[11390, iters=109]	
CSTR50b50c1p02.dat	766.7 [897]	2434.8[1466]	767.4 [4117]	CG842.5 [5631]	842.5[13334, iters=111]	
CSTR50b50c1p03.dat	792.2 [756]	2499 [1147]	810.9 [3437]	CG860.2 [5182]	860.2[11267, iters=104]	
CSTR50b50c1p4.dat	957.4 [736]	2449.4[1093]	957.5 [2850]	CG1116.4[3458]	1116.4[6133, iters=80]	
CSTR50b50c1p5.dat	818.1 [1104]	2414.4[1707]	819.3 [5034]	CG879.1 [6345]	879.05[8723, iters=84]	
CSTR50b50c1p6.dat	783.4 [969]	2454.2[1698]	784.1 [6380]	CG840.1 [8213]	840.02[11097, iters=97]	
CSTR50b50c1p7.dat	911.8 [626]	2466.2[940]	929.2 [3251]	CG994.8 [4397]	994.8[7634, iters=90]	
CSTR50b50c2p01.dat	616.8 [857]	2488 [1470]	626.7 [4533]	CG672.3 [7649]	672.3[18363, iters=129]	
CSTR50b50c2p02.dat	581.5 [1297]	2463.2[2211]	581.6 [5727]	CG593.1 [9155]	593.1[16108, iters=128]	
CSTR50b50c2p03.dat	479.7 [1479]	2333.6[2401]	479.7 [9731]	CG480.1 [16300]	480.047[30205, iters=207]	
CSTR50b50c2p4.dat	539.9 [1719]	2451 [2476]	540 [5814]	CG542 [11316]	541.955[25012, iters=168]	
CSTR50b50c2p5.dat	569.6 [1284]	2377.8[2035]	569.6 [5805]	CG581.6 [8822]	581.567[16901, iters=120]	
CSTR50b50c2p6.dat	517.5 [1416]	2493.3[2352]	517.6 [6765]	CG519.3 [11989]	519.25[22235, iters=155]	
CSTR50b50c2p7.dat	553 [1379]	2476 [2164]	553 [6981]	CG573.4 [12778]	573.359[28417, iters=188]	
CSTR50b50c3p01.dat	282 [1146]	2369 [2855]	282 [9785]	CG282 [21323]	281.949[44997, iters=199]	
CSTR50b50c3p02.dat	239.4 [1628]	2422.4[3288]	239.4 [10945]	CG239.4 [23865]	239.354[42835, iters=179]	
CSTR50b50c3p03.dat	271.4 [1687]	2350.2[3466]	271.4 [11274]	CG271.4 [21618]	271.313[45713, iters=201]	
CSTR50b50c3p4.dat	282.4 [1792]	2297.2[3222]	282.4 [9332]	CG282.4 [17451]	282.334[41107, iters=206]	
CSTR50b50c3p5.dat	269.9 [1612]	2401.8[3140]	269.9 [11543]	CG269.9 [18775]	269.801[41890, iters=209]	
CSTR50b50c3p6.dat	274.8 [1544]	2381 [3149]	274.8 [9140]	CG274.8 [25980]	274.8[43429, iters=198]	
CSTR50b50c3p7.dat	273.2 [1625]	2378 [2809]	273.2 [11882]	CG273.2 [19042]	273.161[37547, iters=202]	
CSTR50b50c4p01.dat	576.7 [678]	2444.3[1327]	576.9 [3568]	CG579.6 [6916]	579.548[20243, iters=159]	
CSTR50b50c4p02.dat	548.3 [918]	2409.6[1588]	548.4 [3793]	CG551.1 [7946]	551.01[21091, iters=163]	
CSTR50b50c4p03.dat	663.6 [802]	2457.4[1330]	665.5 [3117]	CG700.1 [5534]	700.039[15972, iters=151]	
CSTR50b50c4p4.dat	562.9 [1013]	2328.4[1653]	562.9 [4041]	CG564.9 [7394]	564.837[20166, iters=167]	
CSTR50b50c4p5.dat	629.7 [687]	2418.8[1261]	629.9 [3243]	CG672.5 [6352]	672.47[17341, iters=152]	
CSTR50b50c4p6.dat	633.1 [557]	2460.2[1110]	633.2 [2847]	CG642.6 [4424]	642.6[12990, iters=133]	
CSTR50b50c4p7.dat	599.5 [846]	2466 [1624]	599.5 [4575]	CG603 [7096]	602.935[17079, iters=155]	
CSTR50b50c5p01.dat	337.7 [939]	2288.4[1831]	337.7 [9073]	CG337.7 [14128]	337.675[32154, iters=228]	
CSTR50b50c5p02.dat	349.8 [1081]	2339.2[1894]	349.8 [9010]	CG349.8 [13255]	349.701[34329, iters=250]	
CSTR50b50c5p03.dat	295.7 [1248]	2252.4[2191]	295.7 [8434]	CG295.7 [13417]	295.693[26976, iters=191]	
CSTR50b50c5p4.dat	344.9 [1398]	2373.8[2266]	344.9 [11766]	CG344.9 [16425]	344.874[28463, iters=201]	
CSTR50b50c5p5.dat	306.3 [1238]	2428.8[2140]	306.3 [11008]	CG306.3 [15382]	306.263[28735, iters=202]	
CSTR50b50c5p6.dat	340.3 [1259]	2426.8[2145]	340.3 [8952]	CG340.3 [13455]	340.224[32250, iters=223]	
CSTR50b50c5p7.dat	328.3 [1191]	2362.6[2114]	328.3 [9540]	CG328.3 [15448]	328.295[29120, iters=213]	
wascher-1	24 [219]	112.6 [638]	24.1 [3049]	CG24.1 [14176]	24.0648[19869, iters=507]	
wascher-2	19.8 [216]	93.4 [660]	19.8 [2964]	CG22.1 [26445]	22.0003[29747, iters=446]	
wascher-3	12.1 [134]	54.6 [252]	12.1 [2596]	CG12.2 [4280]	12.1219[2079, iters=157]	
wascher-4	23.2 [230]	109.2 [643]	23.2 [3167]	CG23.2 [16776]	23.1987[20661, iters=449]	
wascher-5	12 [218]	156.6 [791]	12 [5372]	CG12 [157357]	11.9991[199695, iters=924]	
wascher-6	9.5 [283]	133.6 [620]	9.5 [5908]	CG9.5 [52014]	9.42343[168343, iters=774]	
wascher-7	12 [321]	135.6 [726]	12 [7316]	CG12 [81479]	11.997[127617, iters=738]	

Instance by instance results for Multiple Length Cutting Stock with $\mu(0.2C)=0.1$, $\mu(0.7C)=0.6$ and $\mu(C)=1$

Instance name/id	k=1		k=2		k=3		ColGen(T_{ms})
	lb [T_{lb}]	ub [T_{ub}]	lb [T_{lb}]	ub [T_{ub}]	lb [T_{lb}]	ub [T_{ub}]	Opt/Lb
'HARD1'	47.5 [1063]	198.6 [5418]	47.7 [72539]	197.2 [76759]	47.9 [77523]	CG51.5 [141610]	51.4253[82831, iters=732]
'HARD2'	47.7 [1080]	198.6 [5534]	47.8 [69778]	197.2 [73881]	47.9 [74605]	CG51.5 [136731]	51.442[80426, iters=727]
'HARD3'	46.8 [1000]	198.6 [5405]	46.9 [75653]	197.2 [79883]	46.9 [80449]	CG50.6 [149160]	50.5923[88043, iters=793]
'HARD4'	47.6 [1070]	198.6 [5212]	47.8 [75174]	197.2 [79289]	48.1 [79859]	CG52.1 [141449]	52.0829[80566, iters=737]
'HARD5'	40 [947]	198.6 [5492]	40.3 [87079]	197.2 [91622]	40.5 [92164]	CG51.5 [159480]	51.4839[87297, iters=779]
'HARD6'	47.6 [1020]	198.6 [5190]	47.7 [79857]	197.2 [83939]	47.8 [84417]	CG51.9 [149045]	51.862[87345, iters=788]
'HARD7'	46.5 [1037]	198.6 [5460]	46.6 [90837]	197.2 [95107]	46.7 [95623]	CG49.2 [167864]	49.1857[94484, iters=822]
m01-1	26.5 [5]	99.1 [14]	39.9 [23]	99.1 [25]	39.9 [35]	CG49.3 [151]	49.3[78, iters=147]
m01-2	29 [1]	99.1 [7]	43.4 [10]	98.7 [12]	46.5 [16]	CG53 [110]	53[71, iters=157]
m01-3	25.5 [1]	99.1 [7]	39 [10]	99.1 [11]	41.9 [16]	CG48.2 [117]	48.2[64, iters=162]
m01-4	28.7 [2]	99.1 [8]	43.6 [12]	99.1 [13]	44.7 [18]	CG53.2 [85]	53.2[52, iters=134]
m01-5	24.3 [1]	98.1 [7]	34.9 [10]	97.7 [12]	39.4 [16]	CG44.3 [148]	44.3[116, iters=195]
m01-6	30.7 [1]	99.1 [7]	43.9 [12]	97.8 [15]	45.8 [19]	CG51.8 [101]	51.8[73, iters=147]
m01-7	26.1 [1]	98.1 [5]	31.9 [7]	96.2 [9]	35.6 [11]	CG50.5 [72]	50.5[86, iters=193]
m20-1	47.4 [5]	99.6 [12]	48.4 [20]	99.6 [22]	48.4 [29]	CG56.6 [112]	56.6[120, iters=198]
m20-2	50 [2]	99.6 [7]	53.2 [10]	99.2 [12]	54.8 [15]	CG58.7 [73]	58.7[65, iters=161]
m20-3	53.1 [2]	97.6 [7]	55.2 [11]	97.6 [12]	55.2 [16]	CG64.8 [58]	64.8[46, iters=120]
m20-4	46.5 [2]	99.6 [7]	50.5 [11]	99.6 [12]	50.5 [15]	CG58.9 [68]	58.9[53, iters=130]
m20-5	50.1 [2]	99.6 [7]	52 [11]	99.6 [12]	52 [16]	CG59.5 [66]	59.5[68, iters=164]
m20-6	50.7 [2]	99.6 [7]	51.5 [11]	99.6 [12]	51.5 [16]	CG60.2 [57]	60.2[54, iters=144]
m20-7	52.3 [2]	99.6 [6]	54.2 [10]	99.6 [11]	54.2 [14]	CG63.9 [51]	63.9[47, iters=119]
m35-1	58.3 [4]	100 [8]	62.6 [14]	100 [16]	62.9 [21]	CG73.9 [66]	73.9[36, iters=68]
m35-2	57.9 [2]	100 [6]	60.5 [10]	100 [11]	60.7 [14]	CG71.5 [61]	71.5[36, iters=88]
m35-3	58.7 [1]	100 [4]	61.9 [7]	100 [8]	62.1 [10]	CG73.7 [30]	73.7[34, iters=96]
m35-4	57.3 [1]	99.6 [5]	60.4 [7]	99.6 [8]	62.3 [10]	CG72.7 [52]	72.7[28, iters=83]
m35-5	58.9 [1]	99 [5]	64.9 [8]	98.6 [9]	65 [11]	CG75.3 [32]	75.3[23, iters=77]
m35-6	58 [1]	99.6 [5]	62.9 [7]	99.2 [8]	63.1 [10]	CG72.2 [30]	72.2[32, iters=111]
m35-7	56.6 [1]	100 [4]	60 [7]	100 [8]	60.5 [10]	CG67.2 [53]	67.2[26, iters=86]
CSTR30p0	47 [192]	672.1 [478]	50.2 [996]	625.6 [1218]	51.2 [1382]	CG63.8 [2064]	63.75[2602, iters=104]
CSTRd43p20	20.2 [6]	130.3 [1328]	22.7 [1364]	116.6 [2691]	22.8 [2710]	101.3 [4479]	24.2292[3069, iters=123]
CSTRd43p21	21.3 [109]	178.2 [2013]	29 [5003]	170.7 [6743]	33.6 [7951]	CG34.1 [13777]	34.05[5985, iters=141]
CSTRd4p1	19 [0]	19 [55]					19[30, iters=5]
CSTR5p11	12.4 [3]	98.5 [1971]	13.4 [1982]	58 [3457]	13.9 [3462]	13.9 [3953]	13.85[1218, iters=16]
CSTR6p20	7 [1]	23.2 [61]	8.8 [67]	8.8 [75]			8.8[19, iters=17]
CSTR7p14	39.3 [5]	161.7 [62]	54.3 [74]	54.3 [90]			54.3[102, iters=21]
CSTR20b50c1p1.dat	172.2 [168]	931.7 [356]	225.8 [1101]	855.2 [1196]	243.1 [1294]	CG255.4 [1507]	255.4[1490, iters=42]
CSTR20b50c1p2.dat	309.6 [8]	934.4 [109]	393.4 [140]	826.6 [273]	396.9 [285]	CG455.5 [403]	455.5[704, iters=30]
CSTR20b50c1p3.dat	181.2 [283]	958.3 [526]	284.4 [2140]	946.3 [2283]	299.9 [2662]	CG314.7 [2970]	314.625[1134, iters=37]
CSTR20b50c1p4.dat	221.3 [176]	880.1 [461]	295.2 [1767]	817.4 [2022]	314.7 [2329]	CG334.4 [2606]	334.35[1254, iters=40]
CSTR20b50c1p5.dat	225.2 [267]	982.1 [472]	282.4 [1841]	802 [2163]	283 [2486]	CG328.4 [2709]	328.4[1476, iters=39]
CSTR20b50c2p1.dat	138.8 [452]	911.1 [748]	197.7 [3901]	804.7 [4198]	204.5 [5112]	CG219.2 [5733]	219.12[2547, iters=55]
CSTR20b50c2p2.dat	91.9 [576]	873.3 [1077]	139.6 [4329]	720.5 [4670]	140.5 [4894]	CG149.7 [5890]	149.6[2911, iters=52]
CSTR50b50c1p01.dat	538.9 [490]	2469.7[962]	744 [4700]	2457.3[5095]	764.7 [8263]	CG866.3 [9615]	866.3[9513, iters=105]
CSTR50b50c1p02.dat	469.8 [917]	2453.3[1633]	706 [4277]	2413.7[4757]	750.2 [5232]	CG842.5 [7193]	842.5[11207, iters=105]
CSTR50b50c1p03.dat	603.8 [705]	2489.3[1247]	725.4 [3313]	2444.1[3839]	755.5 [4297]	CG860.2 [5832]	860.2[10592, iters=100]
CSTR50b50c1p4.dat	569.5 [673]	2429 [927]	874.9 [2620]	2372.9[2862]	916.4 [3435]	CG1116.4[3939]	1116.4[5896, iters=78]
CSTR50b50c1p5.dat	503.8 [982]	2420 [1629]	757.2 [4559]	2308.5[4998]	785.2 [5154]	CG879.1 [6318]	879.05[9116, iters=88]
CSTR50b50c1p6.dat	493.4 [990]	2445.3[1751]	678.9 [10330]	2305 [10999]	737.7 [19534]	CG840.1 [21234]	840.02[9946, iters=97]
CSTR50b50c1p7.dat	597.8 [570]	2488.4[901]	838.9 [3086]	2218.5[3828]	839.1 [4745]	CG994.8 [5533]	994.8[7194, iters=90]
CSTR50b50c2p01.dat	377.7 [814]	2489.3[1563]	633.6 [4520]	2287 [5635]	638.6 [6413]	CG672.3 [9355]	672.3[15436, iters=114]
CSTR50b50c2p02.dat	349.1 [1163]	2444.6[2031]	542.5 [5940]	2365.7[7118]	544.1 [8536]	CG593 [11136]	592.925[15110, iters=123]
CSTR50b50c2p03.dat	293.7 [1142]	2483.9[2037]	417 [5408]	2388.5[6005]	418.1 [7092]	CG448.2 [11697]	448.125[22206, iters=164]
CSTR50b50c2p4.dat	323.7 [1418]	2484.8[2337]	502.4 [5710]	2238.1[7217]	508.2 [8309]	CG534 [11452]	534[19176, iters=141]
CSTR50b50c2p5.dat	363.7 [1178]	2410.1[1978]	545.2 [5837]	2286.9[6402]	545.5 [7377]	CG578.7 [11966]	578.602[22276, iters=152]
CSTR50b50c2p6.dat	304.1 [1290]	2336.5[2211]	442.7 [9298]	2315.8[9815]	471.8 [14891]	CG509.1 [18327]	509.075[17963, iters=138]
CSTR50b50c2p7.dat	325.9 [1334]	2336.4[2240]	396.2 [8314]	2313 [8923]	537.7 [12581]	CG573.4 [18319]	573.359[25396, iters=176]
CSTR50b50c3p01.dat	166 [1086]	2359.7[2742]	171.1 [12882]	2359.7[13439]	171.1 [24638]	CG238.8 [34146]	238.713[38197, iters=200]
CSTR50b50c3p02.dat	140.4 [1503]	2473.1[3361]	166.9 [7461]	2399.5[8681]	167 [11387]	CG185.3 [20879]	185.2[31959, iters=155]
CSTR50b50c3p03.dat	158.4 [1473]	2337.4[3087]	158.7 [9361]	2305.9[10447]	200.2 [15462]	CG217.5 [21989]	217.5[27811, iters=154]
CSTR50b50c3p4.dat	165.1 [1732]	2437.1[3078]	185 [7288]	2410.7[8170]	185.2 [14806]	CG206.8 [21151]	206.8[29670, iters=166]
CSTR50b50c3p5.dat	158 [1559]	2483.9[3236]	190.6 [7815]	2389.7[9305]	190.8 [12786]	CG209.8 [20066]	209.8[28745, iters=158]
CSTR50b50c3p6.dat	161.2 [1506]	2407.2[3030]	163 [9230]	2397.1[10255]	163.1 [18019]	CG225.6 [24305]	225.55[27173, iters=152]
CSTR50b50c3p7.dat	163.2 [1518]	2454 [2886]	164.3 [14832]	2254.9[16209]	184.3 [30073]	CG201 [36378]	200.956[31718, iters=189]
CSTR50b50c4p01.dat	373.6 [695]	2500.1[1335]	543.8 [4138]	2224.6[5226]	544.3 [14353]	CG567.1 [16879]	567.09[17631, iters=146]
CSTR50b50c4p02.dat	346.6 [821]	2438 [1476]	499.5 [3784]	2191.7[4732]	500.6 [5726]	CG526.3 [9014]	526.25[19926, iters=160]
CSTR50b50c4p03.dat	537 [704]	2433.5[1146]	664.5 [3140]	2160.2[4029]	666.1 [4717]	CG700.1 [6932]	700.039[15888, iters=149]
CSTR50b50c4p4.dat	371.2 [1075]	2468.6[1715]	516.5 [4452]	2211.5[5328]	517.4 [5905]	CG548.7 [8445]	548.625[18451, iters=152]
CSTR50b50c4p5.dat	484.5 [704]	2500.1[1193]	622.9 [3402]	2237.2[4336]	624.8 [4707]	CG672.5 [7438]	672.47[16856, iters=151]
CSTR50b50c4p6.dat	483.3 [574]	2460.5[955]	610.6 [3324]	2242.2[4093]	611.2 [5112]	CG637.8 [6806]	637.725[13871, iters=139]
CSTR50b50c4p7.dat	465.8 [737]	2462.3[1235]	565.7 [3569]	2311.9[4319]	569.2 [5157]	CG588.5 [7615]	588.45[17058, iters=152]
CSTR50b50c5p01.dat	200 [866]	2381.3[2138]	210.1 [6162]	2157.3[7122]	210.2 [12898]	CG277.1 [17847]	277.037[28115, iters=200]
CSTR50b50c5p02.dat	204.6 [969]	2317.5[2088]	204.8 [10376]	2258.7[11215]	205.1 [20923]	CG294 [26178]	293.9[24195, iters=177]
CSTR50b50c5p03.dat	172.9 [1163]	2492 [2168]	217.9 [5276]	2290.1[6294]	218 [6659]	CG230.2 [12095]	230.2[26260, iters=185]
CSTR50b50c5p4.dat	202.7 [1266]	2419.4[2475]	202.7 [12507]	2256.8[13419]	204.4 [27555]	CG276.5 [32279]	276.494[23308, iters=170]
CSTR50b50c5p5.dat	179 [1164]	2419.1[2154]	209.4 [5454]	2356.2[6439]	210 [6642]	CG239 [11520]	238.938[27927, iters=201]
CSTR50b50c5p6.dat	201.3 [1177]	2440.7[2150]	270.5 [5780]	2433.5[6437]	271.6 [7239]	CG283.5 [12374]	283.452[26099, iters=184]
CSTR50b50c5p7.dat	191.6 [1002]	2400.9[1964]	191.7 [6390]	2284.1[7354]	245.6 [11721]	CG263.7 [16733]	263.7[22830, iters=168]

Table 2: More Detailed Results

Cutting Stock instance by instance results

In brackets, we provide the total (lower and, respectively, upper bounding) time in milliseconds. The key word CG indicates that the final Column Generation phase has been executed in the end, as the bounds lb and ub were already very close (conditions described in the paper).

Instance name/Id	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	ColGen (ms)							
	lb [T _{lb}]	ub [T _{ub}]	lb [T _{lb}]	ub [T _{ub}]	lb [T _{lb}]	ub [T _{ub}]	lb [T _{lb}]	ub [T _{ub}]	lb [T _{lb}]	ub [T _{ub}]	Opt/Lb							
HARD1	55.388[380]	[380]	56.037[8399]	56.989[27065]							56.039[77960,items=655]							
HARD2	55.558[252]	[252]	55.978[9834]	CG55.999[32683]							55.981[77746,items=665]							
HARD3	54.416[261]	[261]	54.922[5383]	CG54.999[28100]							54.923[79483,items=672]							
HARD4	55.507[251]	[251]	56.408[5866]	CG56.913[34104]							56.436[71173,items=628]							
HARD5	54.808[261]	[261]	55.523[7084]	55.994[27177]							55.523[75361,items=657]							
HARD6	55.445[253]	[253]	56.096[7945]	56.884[26351]							56.896[571733,items=630]							
HARD7	53.992[274]	[274]	54.198[5068]	57.406[31739]	54.198[53138]	CG55 [53630]					54.248[78884,items=743]							
HARD8	55.54 [261]	[261]	56.088[9748]	56.94 [28356]							56.083[72737,items=632]							
HARD9	55.005[262]	[262]	55.473[6948]	55.933[27088]							55.473[78066,items=670]							
HARD0	54.403[265]	[265]	55.001[4545]	55.999[24724]							55.006[80926,items=686]							
m01-1	48.62 [5]	[5]	54 [8]	54 [96]							54 [117,items=111]							
m01-2	52.309[2]	[2]	52.309[5]	77 [24]	55 [25]	55 [49]					55 [88,items=134]							
m01-3	49.39 [1]	[1]	49.39 [4]	54 [199]	54 [200]	54 [200]					54 [81,items=112]							
m01-4	52.643[1]	[1]	56 [2]	56 [69]							56 [75,items=134]							
m01-5	44.72 [1]	[1]	44.72 [3]	CG45.25 [368]							45.25 [206,items=204]							
m01-6	52.041[1]	[1]	56 [2]	60 [35]	58 [36]	58 [37]					58 [89,items=108]							
m01-7	50.459[1]	[1]	50.464[2]	61.5 [40]	50.464[1]	61.5 [41]	53 [42]	53 [60]			53 [84,items=153]							
m01-8	52.969[1]	[1]	54 [2]	56 [58]	54 [59]	56 [59]	54.667[61]	55 [63]			54.714[3] 75,items=181]							
m01-9	49.77 [1]	[1]	49.77 [2]	51.5 [165]	49.77 [167]	51.5 [167]	51.5 [168]	51.5 [168]			51.5 [85,items=175]							
m01-10	54.418[1]	[1]	55 [2]	60 [46]	55 [47]	60 [47]	57.5 [49]	58 [51]			57.5 [61,items=127]							
m20-1	55.347[4]	[4]	55.371[9]	63.435[80]	55.492[82]	62.784[83]	58.667[86]	59 [126]			59 [95,items=148]							
m20-2	57.756[1]	[1]	57.756[3]	80 [16]	58.379[18]	CG61 [43]					61 [61,items=149]							
m20-3	62.092[1]	[1]	64 [3]	72 [62]	68 [64]	68 [67]					68 [38,items=106]							
m20-4	56.978[1]	[1]	56.978[3]	80 [17]	56.978[17]	CG60.5 [40]					60.5 [51,items=128]							
m20-5	58.541[1]	[1]	58.541[4]	79 [15]	58.557[17]	CG62.5 [37]					62.5 [37,items=99]							
m20-6	59.427[1]	[1]	61 [3]	67 [78]	65 [80]	65 [82]					65 [39,items=110]							
m20-7	61.204[1]	[1]	61.204[3]	74.5 [18]	61.797[19]	74.5 [19]	65.5 [21]	CG66.5 [36]			66.5 [32,items=96]							
m20-8	57.327[1]	[1]	57.327[3]	78.5 [16]	57.327[3]	61 [66]					60.9 [48,items=125]							
m20-9	56.796[1]	[1]	56.796[3]	78 [15]	56.796[16]	78 [16]	60 [17]	CG61 [37]			61 [42,items=113]							
m20-10	60.429[1]	[1]	60.429[3]	82 [14]	60.429[15]	CG64 [38]					64 [48,items=128]							
m35-1	66.827[4]	[4]	74.431[8]	85 [22]	74.431[25]	CG77.5 [39]					77.5 [28,items=55]							
m35-2	66 [2]	[2]	75.214[5]	88 [17]	77 [19]	CG77.5 [52]					77.5 [23,items=54]							
m35-3	67.653[2]	[2]	73 [4]	86 [38]	80.333[40]	CG80.5 [43]					80.5 [21,items=53]							
m35-4	66.194[1]	[1]	75.3 [3]	76 [35]							75.3 [22,items=63]							
m35-5	68.51 [1]	[1]	73.3 [2]	88 [11]	80.5 [12]	81 [17]					80.5 [18,items=55]							
m35-6	65.677[1]	[1]	67.281[3]	92 [8]	77 [10]	77 [34]					77 [20,items=68]							
m35-7	63.296[1]	[1]	70 [3]	70 [73]							70 [20,items=77]							
m35-8	66.051[1]	[1]	80 [2]	84 [38]	82.5 [40]	83 [40]					82.5 [13,items=47]							
m35-9	64.854[1]	[1]	76 [34]	76 [34]							75.5 [23,items=68]							
m35-10	63.592[1]	[1]	66.096[3]	89.5 [10]	66.096[11]	CG71.5 [31]					71.5 [22,items=82]							
vbInd-30p0	89.448[179]	[179]	89.448[687]	89.819[1567]							89.4478 [2384,items=96]							
vbInd-443p20	28.515[3]	[3]	28.515[11]	35.769[2275]	28.515[281]	35.769[2321]	28.521[2332]	33.731[3084]	28.525[3095]	33.697[3209]	28.534[3218]	33.289[3596]	28.535[3603]	32.66 [4126]	28.542[4136]	32.66 [4126]	28.55 [4222]	28.862[4381]
vbInd-443p21	39.507[32]	[32]	39.507[1020]	68.69[2905]	39.508[492]	CG39.939[6245]					28.5503 [1953,items=728]	39.5086 [6185,items=174]						
vbInd-44p1	19 [0]	[0]	19 [0]	19 [25]							19 [3,items=5]							
vbInd-5p11	20.536[1]	[1]	20.539[2]	41 [649]	20.625[650]	20.627[757]					20.625 [936,items=8]							
vbInd-6p20	12.501[0]	[0]	12.505[2]	24.667[9]	12.525[11]	CG12.671[19]					12.5368 [23,items=15]							
vbInd-7p14	69.461[5]	[5]	69.469[11]	138.62[47]	69.495[56]	CG69.682[75]					69.5877 [90,items=15]							
vbInd-7p17	8.7373[96]	[96]	8.7373[766]	8.8857[897]							8.7373 [655,items=17]							
vbInd-7p18	90.966[11]	[11]	90.978[27]	150 [75]	90.188[81]	CG90.902[123]					90.902 [138,items=14]							
vbInd-11p4	100.91[75]	[75]	100.91[1560]	100.91[12250]							100.91[17606,items=25]							
vbInd-12p19	22.887[12]	[12]	22.887[60]	26.345[142]	22.888[163]	CG22.929[206]					22.8953 [160,items=31]							
vbInd-12p24	57.42 [52]	[52]	57.42 [129]	66.688[327]	57.42 [338]	CG57.612[354]					57.4575 [514,items=40]							
vbInd-3b2	55.093[31]	[31]	55.093[74]	55.923[6802]							55.0929 [10262,items=35]							
vbInd-18p22	32.027[35]	[35]	32.027[256]	32.986[403]							32.0274 [728,items=63]							
vbInd-25p0	114.89[122]	[122]	114.89[263]	CG114.91[1263]							114.887 [2010,items=95]							
vbInd-28p0	187.73[275]	[275]	187.73[655]	187.98[2043]							187.731 [3988,items=81]							
vbInd-416p6	36.912[0]	[0]	36.912[1]	62 [107]	36.93 [108]	56.2 [171]	36.95 [172]	56.2 [182]	37.104[183]	37.7 [236]	37.1042 [311,items=32]							
vb20c1p1.dat	294.94[115]	[115]	295.02[225]	714.5 [467]	295.02[225]	714.5 [467]	CG295.85[799]				295.512 [1698,items=42]							
vb20c1p2.dat	459.23[6]	[6]	460.62[12]	839 [194]	471 [195]	476 [975]					473.5 [833,items=31]							
vb20c1p3.dat	344.67[105]	[105]	344.76[316]	812 [570]	345.68[582]	CG347.97[743]					347.967 [1477,items=33]							
vb20c1p4.dat	377.34[86]	[86]	377.44[190]	800 [419]	377.34[86]	CG394 [729]					377.134 [604,items=35]							
vb20c1p5.dat	353.72[141]	[141]	354.46[306]	739.5 [553]	354.68[676]	CG361.1 [913]					361.1 [1606,items=34]							
vb20c2p1.dat	248.61[219]	[219]	248.63[462]	682.5 [798]	248.66[862]	CG249.37[1437]					249.366 [3016,items=54]							
vb20c2p2.dat	177.86[377]	[377]	177.86[764]	CG177.96[4884]							177.869 [7253,items=84]							
vb20c2p3.dat	377.35[7]	[7]	377.44[17]	916.5 [233]	382.41[238]	597.25[593]	383.1 [595]	421.75[737]	383.18[740]	418.17[846]	383.29[848]	385.75[989]	383.29[1005]	383.71[1111]				
vb20c2p4.dat	261.43[145]	[145]	261.45[383]	647 [700]	261.46[759]	CG261.95[1034]					261.659 [2865,items=58]							
vb20c2p5.dat	235.35[224]	[224]	235.35[463]	296.5 [2181]	235.35[2240]	296.5 [2244]	CG235.95[4081]				235.411 [3420,items=59]							
vb20c3p1.dat	146.34[270]	[270]	146.34[1316]	146.82[2667]							146.344 [3849,items=64]							
vb20c3p2.dat	104.42[429]	[429]	104.42[1056]	104.42[3016]							104.415 [4826,items=54]							
vb20c3p3.dat	141.51[367]	[367]	141.51[1387]	141.87[3288]							141.512 [5554,items=73]							
vb20c3p4.dat	147.51[385]	[385]	147.51[1064]	147.96[3201]							147.512 [6336,items=76]							
vb20c3p5.dat	71.776[532]	[532]	71.776[385]	71.79 [3935]							71.776 [604,items=50]							
vb20c4p1.dat	228.83[160]	[160]	228.84[414]	354 [1171]	228.84[1242]	CG228.99[1516]					228.905 [2942,items=63]							
vb20c4p2.dat	275.33[118]	[118]	275.34[260]	669 [512]	275.4 [542]	CG276.09[841]					275.092 [2127,items=49]							
vb20c4p3.dat	254.42[161]	[161]	254.43[340]	686 [612]	254.45[836]	CG254.95[1148]					254.09 [2809,items=59]							
vb20c4p4.dat	229.95[130]	[130]	229.96[317]	736.25[579]	229.97[647]	CG230.01[1009]					230.009 [2680,items=64]							
vb20c4p5.dat	242.59[121]	[121]	242.59[264]	379 [949]	242.61[1132]	CG243.39[1425]					243.387 [2008,items=47]							
vb20c5p1.dat	116.02[328]	[328]	116.02[1384]	116.97[2472]							116.018 [3869,items=62]							
vb20c5p2.dat	139.98[281]	[281]	139.98[1673]	139.98[2914]							139.978 [3845,items=64]							
vb20c5p3.dat	143.87[255]	[255]	143.87[1849]	143.87[3104]							143.871 [3862,items=74]							
vb20c5p4.dat	174.67[174]	[174]	174.67[365]	181.25[1738]							174.678 [3940,items=89]							
vb20c5p5.dat	132.32[286]	[286]	132.32[1967]	132.94[3147]							132.319 [3658,items=71]							
vb50c1p01.dat	935.56[120]	[120]	935.56[831]	2235 [1470]	935.58[1856]	CG938.24[3200]					938.638 [407,items=64]							
vb50c1p02.dat	893.51[644]	[644]	893.63[1306]	2061 [2253]	893.66[3221]	CG897.1 [5521]					897.102 [17530,items=123]							
vb50c1p03.dat	925.1 [496]	[496]	925.14[1045]	2287 [1717]	925.32[3254]	CG927.07[4396]					927.07 [18084,items=142]							
vb50c1p04.dat	1118.4[289]	[289]	1119.2[495]	2258 [1035]	1221 [1049]	CG1125 [7732]												

Instance by instance results for Multiple Length Cutting Stock with $\mu(0.7C)=0.6$ and $\mu(C)=1$						
Instance name/id	k=1		k=2		ColGen(T_{ms})	
	lb [T_{lb}]	ub [T_{ub}]	lb [T_{lb}]	ub [T_{ub}]	Opt/Lb	
'HARD1'	47.5 [941]	198.6 [5290]	47.7 [70579]	CG51.5 [133995]	51.4253[84540,itors=746]	
'HARD2'	47.7 [1015]	198.6 [5114]	47.8 [69170]	CG51.5 [129665]	51.442[82495,itors=741]	
'HARD3'	46.8 [972]	198.6 [5156]	46.9 [70460]	CG50.6 [134708]	50.5923[89323,itors=788]	
'HARD4'	47.6 [963]	198.6 [5060]	47.8 [68995]	CG52.1 [126929]	52.0829[81661,itors=737]	
'HARD5'	47.1 [1060]	198.6 [5231]	47.3 [79083]	CG51.5 [143326]	51.484[89721,itors=791]	
'HARD6'	47.6 [925]	198.6 [5023]	47.7 [78906]	CG51.9 [140635]	51.862[85698,itors=760]	
'HARD7'	46.5 [1106]	198.6 [5418]	46.6 [89637]	CG49.2 [160297]	49.1857[98024,itors=828]	
m01-1	41.7 [6]	93.6 [15]	41.7 [22]	CG49.3 [153]	49.3[128,itors=139]	
m01-2	45.7 [2]	99.6 [8]	48.5 [11]	CG53 [105]	53[97,itors=166]	
m01-3	42.4 [2]	95.6 [8]	42.4 [12]	CG48.2 [110]	48.2[85,itors=161]	
m01-4	45.1 [2]	95.6 [8]	45.1 [12]	CG53.2 [79]	53.2[64,itors=125]	
m01-5	38.6 [2]	99.6 [7]	40.5 [10]	CG44.3 [149]	44.3[177,itors=212]	
m01-6	45 [2]	99.6 [8]	47.3 [12]	CG51.8 [113]	51.8[94,itors=147]	
m01-7	43.3 [1]	95.6 [5]	43.3 [8]	CG50.5 [68]	50.5[86,itors=167]	
m20-1	47.4 [5]	98.6 [12]	48.4 [20]	CG56.6 [106]	56.6[111,itors=172]	
m20-2	50 [2]	99.6 [7]	53.2 [10]	CG58.7 [76]	58.7[62,itors=158]	
m20-3	53.1 [2]	97.6 [7]	55.2 [11]	CG64.8 [54]	64.8[44,itors=123]	
m20-4	48.4 [2]	97.6 [7]	50.5 [11]	CG58.9 [62]	58.9[57,itors=133]	
m20-5	50.1 [2]	97.6 [7]	50.1 [12]	CG59.5 [62]	59.5[67,itors=161]	
m20-6	50.7 [2]	97.6 [7]	50.7 [11]	CG60.2 [58]	60.2[44,itors=125]	
m20-7	52.3 [2]	97.6 [6]	52.3 [10]	CG63.9 [49]	63.9[46,itors=119]	
m35-1	58.3 [3]	100 [6]	62.6 [10]	CG73.9 [45]	73.9[33,itors=68]	
m35-2	57.9 [2]	100 [6]	60.5 [9]	CG71.5 [53]	71.5[36,itors=96]	
m35-3	58.7 [1]	100 [4]	61.9 [7]	CG73.7 [28]	73.7[28,itors=89]	
m35-4	57.3 [1]	98.8 [6]	60.6 [8]	CG72.7 [37]	72.7[27,itors=83]	
m35-5	58.9 [1]	99 [5]	64.9 [7]	CG75.3 [31]	75.3[19,itors=73]	
m35-6	58 [1]	99.6 [5]	62.9 [7]	CG72.2 [26]	72.2[26,itors=103]	
m35-7	56.6 [1]	100 [4]	60 [7]	CG67.2 [50]	67.2[25,itors=86]	
CSTR30p0	76.7 [240]	703 [527]	76.7 [1773]	CG76.7 [2575]	76.6757[3176,itors=133]	
CSTRd43p20	24.5 [6]	112.8 [2217]	24.5 [2250]	CG24.6 [4332]	24.5333[3035,itors=121]	
CSTRd43p21	33.9 [178]	153.4 [4818]	33.9 [7591]	CG34.2 [10094]	34.1531[6717,itors=142]	
CSTRd4p1	19 [0]	19 [55]			19[29,itors=5]	
CSTR5p11	17.7 [3]	39.4 [1972]	17.8 [1983]	CG18 [3008]	17.9062[1126,itors=15]	
CSTR6p20	10.8 [3]	42.6 [29]	10.8 [42]	CG10.8 [62]	10.7818[20,itors=18]	
CSTR7p14	59.7 [7]	250.4 [39]	60.2 [66]	CG60.9 [87]	60.885[82,itors=16]	
CSTR20b50c1p1.dat	252.9 [234]	938.6 [422]	254.9 [1649]	CG265.6 [1807]	265.55[1293,itors=36]	
CSTR20b50c1p2.dat	397.7 [9]	933.8 [143]	398.3 [178]	CG455.5 [266]	455.5[706,itors=30]	
CSTR20b50c1p3.dat	297.3 [284]	914 [499]	299.4 [2123]	CG314.7 [2418]	314.625[1140,itors=37]	
CSTR20b50c1p4.dat	323.5 [193]	967.8 [375]	324.9 [1360]	CG337.2 [1698]	337.175[1173,itors=37]	
CSTR20b50c1p5.dat	302.6 [359]	1000.2[564]	306.3 [2073]	CG328.6 [2323]	328.6[1429,itors=36]	
CSTR20b50c2p1.dat	213.4 [472]	910.3 [799]	214.7 [2803]	CG219.3 [3300]	219.3[2633,itors=57]	
CSTR20b50c2p2.dat	152.5 [609]	847.4 [1038]	152.5 [4204]	CG152.6 [5767]	152.578[4885,itors=74]	
CSTR50b50c1p01.dat	802.7 [501]	2453.8[1091]	803.4 [3148]	CG866.3 [4222]	866.3[11390,itors=109]	
CSTR50b50c1p02.dat	766.7 [897]	2434.8[1466]	767.4 [4117]	CG842.5 [5631]	842.5[13334,itors=111]	
CSTR50b50c1p03.dat	792.2 [756]	2499 [1147]	810.9 [3437]	CG860.2 [5182]	860.2[11267,itors=104]	
CSTR50b50c1p4.dat	957.4 [736]	2449.4[1093]	957.5 [2850]	CG1116.4[3458]	1116.4[6133,itors=80]	
CSTR50b50c1p5.dat	818.1 [1104]	2414.4[1707]	819.3 [5034]	CG879.1 [6345]	879.05[8723,itors=84]	
CSTR50b50c1p6.dat	783.4 [969]	2454.2[1698]	784.1 [6380]	CG840.1 [8213]	840.02[11097,itors=97]	
CSTR50b50c1p7.dat	911.8 [626]	2466.2[940]	929.2 [3251]	CG994.8 [4397]	994.8[7634,itors=90]	
CSTR50b50c2p01.dat	616.8 [857]	2488 [1470]	626.7 [4533]	CG672.3 [7649]	672.3[18363,itors=129]	
CSTR50b50c2p02.dat	581.5 [1297]	2463.2[2211]	581.6 [5727]	CG593.1 [9155]	593.1[16108,itors=128]	
CSTR50b50c2p03.dat	479.7 [1479]	2333.6[2401]	479.7 [9731]	CG480.1 [16300]	480.047[30205,itors=207]	
CSTR50b50c2p4.dat	539.9 [1719]	2451 [2476]	540 [5814]	CG542 [11316]	541.955[25012,itors=168]	
CSTR50b50c2p5.dat	569.6 [1284]	2377.8[2035]	569.6 [5805]	CG581.6 [8822]	581.567[16901,itors=120]	
CSTR50b50c2p6.dat	517.5 [1416]	2493.3[2352]	517.6 [6765]	CG519.3 [11989]	519.25[22235,itors=155]	
CSTR50b50c2p7.dat	553 [1379]	2476 [2164]	553 [6981]	CG573.4 [12778]	573.359[28417,itors=188]	
CSTR50b50c3p01.dat	282 [1146]	2369 [2855]	282 [9785]	CG282 [21323]	281.949[44997,itors=199]	
CSTR50b50c3p02.dat	239.4 [1628]	2422.4[3288]	239.4 [10945]	CG239.4 [23865]	239.354[42835,itors=179]	
CSTR50b50c3p03.dat	271.4 [1687]	2350.2[3466]	271.4 [11274]	CG271.4 [21618]	271.313[45713,itors=201]	
CSTR50b50c3p4.dat	282.4 [1792]	2297.2[3222]	282.4 [9332]	CG282.4 [17451]	282.334[41107,itors=206]	
CSTR50b50c3p5.dat	269.9 [1612]	2401.8[3140]	269.9 [11543]	CG269.9 [18775]	269.801[41890,itors=209]	
CSTR50b50c3p6.dat	274.8 [1544]	2381 [3149]	274.8 [9140]	CG274.8 [25980]	274.8[43429,itors=198]	
CSTR50b50c3p7.dat	273.2 [1625]	2378 [2809]	273.2 [11882]	CG273.2 [19042]	273.161[37547,itors=202]	
CSTR50b50c4p01.dat	576.7 [678]	2444.3[1327]	576.9 [3568]	CG579.6 [6916]	579.548[20243,itors=159]	
CSTR50b50c4p02.dat	548.3 [918]	2409.6[1588]	548.4 [3793]	CG551.1 [7946]	551.01[21091,itors=163]	
CSTR50b50c4p03.dat	663.6 [802]	2457.4[1330]	665.5 [3117]	CG700.1 [5534]	700.039[15972,itors=151]	
CSTR50b50c4p4.dat	562.9 [1013]	2328.4[1653]	562.9 [4041]	CG564.9 [7394]	564.837[20166,itors=167]	
CSTR50b50c4p5.dat	629.7 [687]	2418.8[1261]	629.9 [3243]	CG672.5 [6352]	672.47[17341,itors=152]	
CSTR50b50c4p6.dat	633.1 [557]	2460.2[1110]	633.2 [2847]	CG642.6 [4424]	642.6[12990,itors=133]	
CSTR50b50c4p7.dat	599.5 [846]	2466 [1624]	599.5 [4575]	CG603 [7096]	602.935[17079,itors=155]	
CSTR50b50c5p01.dat	337.7 [939]	2288.4[1831]	337.7 [9073]	CG337.7 [14128]	337.675[32154,itors=228]	
CSTR50b50c5p02.dat	349.8 [1081]	2339.2[1894]	349.8 [9010]	CG349.8 [13255]	349.701[34329,itors=250]	
CSTR50b50c5p03.dat	295.7 [1248]	2252.4[2191]	295.7 [8434]	CG295.7 [13417]	295.693[26976,itors=191]	
CSTR50b50c5p4.dat	344.9 [1398]	2373.8[2266]	344.9 [11766]	CG344.9 [16425]	344.874[28463,itors=201]	
CSTR50b50c5p5.dat	306.3 [1238]	2428.8[2140]	306.3 [11008]	CG306.3 [15382]	306.263[28735,itors=202]	
CSTR50b50c5p6.dat	340.3 [1259]	2426.8[2145]	340.3 [8952]	CG340.3 [13455]	340.224[32250,itors=223]	
CSTR50b50c5p7.dat	328.3 [1191]	2362.6[2114]	328.3 [9540]	CG328.3 [15448]	328.295[29120,itors=213]	
wascher-1	24 [219]	112.6 [638]	24.1 [3049]	CG24.1 [14176]	24.0648[19869,itors=507]	
wascher-2	19.8 [216]	93.4 [660]	19.8 [2964]	CG22.1 [26445]	22.0003[29747,itors=446]	
wascher-3	12.1 [134]	54.6 [252]	12.1 [2596]	CG12.2 [4280]	12.1219[2079,itors=157]	
wascher-4	23.2 [230]	109.2 [643]	23.2 [3167]	CG23.2 [16776]	23.1987[20661,itors=449]	
wascher-5	12 [218]	156.6 [791]	12 [5372]	CG12 [157357]	11.9991[199695,itors=924]	
wascher-6	9.5 [283]	133.6 [620]	9.5 [5908]	CG9.5 [52014]	9.42343[168343,itors=774]	
wascher-7	12 [321]	135.6 [726]	12 [7316]	CG12 [81479]	11.997[127617,itors=738]	

Instance by instance results for Multiple Length Cutting Stock with $\mu(0.2C)=0.1$, $\mu(0.7C)=0.6$ and $\mu(C)=1$

Instance name/id	k=1		k=2		k=3		ColGen(T_{ms})
	lb [T_{lb}]	ub [T_{ub}]	lb [T_{lb}]	ub [T_{ub}]	lb [T_{lb}]	ub [T_{ub}]	Opt/Lb
'HARD1'	47.5 [1063]	198.6 [5418]	47.7 [72539]	197.2 [76759]	47.9 [77523]	CG51.5 [141610]	51.4253[82831, iters=732]
'HARD2'	47.7 [1080]	198.6 [5534]	47.8 [69778]	197.2 [73881]	47.9 [74605]	CG51.5 [136731]	51.442[80426, iters=727]
'HARD3'	46.8 [1000]	198.6 [5405]	46.9 [75653]	197.2 [79883]	46.9 [80449]	CG50.6 [149160]	50.5923[88043, iters=793]
'HARD4'	47.6 [1070]	198.6 [5212]	47.8 [75174]	197.2 [79289]	48.1 [79859]	CG52.1 [141449]	52.0829[80566, iters=737]
'HARD5'	40 [947]	198.6 [5492]	40.3 [87079]	197.2 [91622]	40.5 [92164]	CG51.5 [159480]	51.4839[87297, iters=779]
'HARD6'	47.6 [1020]	198.6 [5190]	47.7 [79857]	197.2 [83939]	47.8 [84417]	CG51.9 [149045]	51.862[87345, iters=788]
'HARD7'	46.5 [1037]	198.6 [5460]	46.6 [90837]	197.2 [95107]	46.7 [95623]	CG49.2 [167864]	49.1857[94484, iters=822]
m01-1	26.5 [5]	99.1 [14]	39.9 [23]	99.1 [25]	39.9 [35]	CG49.3 [151]	49.3[78, iters=147]
m01-2	29 [1]	99.1 [7]	43.4 [10]	98.7 [12]	46.5 [16]	CG53 [110]	53[71, iters=157]
m01-3	25.5 [1]	99.1 [7]	39 [10]	99.1 [11]	41.9 [16]	CG48.2 [117]	48.2[64, iters=162]
m01-4	28.7 [2]	99.1 [8]	43.6 [12]	99.1 [13]	44.7 [18]	CG53.2 [85]	53.2[52, iters=134]
m01-5	24.3 [1]	98.1 [7]	34.9 [10]	97.7 [12]	39.4 [16]	CG44.3 [148]	44.3[116, iters=195]
m01-6	30.7 [1]	99.1 [7]	43.9 [12]	97.8 [15]	45.8 [19]	CG51.8 [101]	51.8[73, iters=147]
m01-7	26.1 [1]	98.1 [5]	31.9 [7]	96.2 [9]	35.6 [11]	CG50.5 [72]	50.5[86, iters=193]
m20-1	47.4 [5]	99.6 [12]	48.4 [20]	99.6 [22]	48.4 [29]	CG56.6 [112]	56.6[120, iters=198]
m20-2	50 [2]	99.6 [7]	53.2 [10]	99.2 [12]	54.8 [15]	CG58.7 [73]	58.7[65, iters=161]
m20-3	53.1 [2]	97.6 [7]	55.2 [11]	97.6 [12]	55.2 [16]	CG64.8 [58]	64.8[46, iters=120]
m20-4	46.5 [2]	99.6 [7]	50.5 [11]	99.6 [12]	50.5 [15]	CG58.9 [68]	58.9[53, iters=130]
m20-5	50.1 [2]	99.6 [7]	52 [11]	99.6 [12]	52 [16]	CG59.5 [66]	59.5[68, iters=164]
m20-6	50.7 [2]	99.6 [7]	51.5 [11]	99.6 [12]	51.5 [16]	CG60.2 [57]	60.2[54, iters=144]
m20-7	52.3 [2]	99.6 [6]	54.2 [10]	99.6 [11]	54.2 [14]	CG63.9 [51]	63.9[47, iters=119]
m35-1	58.3 [4]	100 [8]	62.6 [14]	100 [16]	62.9 [21]	CG73.9 [66]	73.9[36, iters=68]
m35-2	57.9 [2]	100 [6]	60.5 [10]	100 [11]	60.7 [14]	CG71.5 [61]	71.5[36, iters=88]
m35-3	58.7 [1]	100 [4]	61.9 [7]	100 [8]	62.1 [10]	CG73.7 [30]	73.7[34, iters=96]
m35-4	57.3 [1]	99.6 [5]	60.4 [7]	99.6 [8]	62.3 [10]	CG72.7 [52]	72.7[28, iters=83]
m35-5	58.9 [1]	99 [5]	64.9 [8]	98.6 [9]	65 [11]	CG75.3 [32]	75.3[23, iters=77]
m35-6	58 [1]	99.6 [5]	62.9 [7]	99.2 [8]	63.1 [10]	CG72.2 [30]	72.2[32, iters=111]
m35-7	56.6 [1]	100 [4]	60 [7]	100 [8]	60.5 [10]	CG67.2 [53]	67.2[26, iters=86]
CSTR30p0	47 [192]	672.1 [478]	50.2 [996]	625.6 [1218]	51.2 [1382]	CG63.8 [2064]	63.75[2602, iters=104]
CSTRd43p20	20.2 [6]	130.3 [1328]	22.7 [1364]	116.6 [2691]	22.8 [2710]	101.3 [4479]	24.2292[3069, iters=123]
CSTRd43p21	21.3 [109]	178.2 [2013]	29 [5003]	170.7 [6743]	33.6 [7951]	CG34.1 [13777]	34.05[5985, iters=141]
CSTRd4p1	19 [0]	19 [55]					19[30, iters=5]
CSTR5p11	12.4 [3]	98.5 [1971]	13.4 [1982]	58 [3457]	13.9 [3462]	13.9 [3953]	13.85[1218, iters=16]
CSTR6p20	7 [1]	23.2 [61]	8.8 [67]	8.8 [75]			8.8[19, iters=17]
CSTR7p14	39.3 [5]	161.7 [62]	54.3 [74]	54.3 [90]			54.3[102, iters=21]
CSTR20b50c1p1.dat	172.2 [168]	931.7 [356]	225.8 [1101]	855.2 [1196]	243.1 [1294]	CG255.4 [1507]	255.4[1490, iters=42]
CSTR20b50c1p2.dat	309.6 [8]	934.4 [109]	393.4 [140]	826.6 [273]	396.9 [285]	CG455.5 [403]	455.5[704, iters=30]
CSTR20b50c1p3.dat	181.2 [283]	958.3 [526]	284.4 [2140]	946.3 [2283]	299.9 [2662]	CG314.7 [2970]	314.625[1134, iters=37]
CSTR20b50c1p4.dat	221.3 [176]	880.1 [461]	295.2 [1767]	817.4 [2022]	314.7 [2329]	CG334.4 [2606]	334.35[1254, iters=40]
CSTR20b50c1p5.dat	225.2 [267]	982.1 [472]	282.4 [1841]	802 [2163]	283 [2486]	CG328.4 [2709]	328.4[1476, iters=39]
CSTR20b50c2p1.dat	138.8 [452]	911.1 [748]	197.7 [3901]	804.7 [4198]	204.5 [5112]	CG219.2 [5733]	219.12[2547, iters=55]
CSTR20b50c2p2.dat	91.9 [576]	873.3 [1077]	139.6 [4329]	720.5 [4670]	140.5 [4894]	CG149.7 [5890]	149.6[2911, iters=52]
CSTR50b50c1p01.dat	538.9 [490]	2469.7[962]	744 [4700]	2457.3[5095]	764.7 [8263]	CG866.3 [9615]	866.3[9513, iters=105]
CSTR50b50c1p02.dat	469.8 [917]	2453.3[1633]	706 [4277]	2413.7[4757]	750.2 [5232]	CG842.5 [7193]	842.5[11207, iters=105]
CSTR50b50c1p03.dat	603.8 [705]	2489.3[1247]	725.4 [3313]	2444.1[3839]	755.5 [4297]	CG860.2 [5832]	860.2[10592, iters=100]
CSTR50b50c1p4.dat	569.5 [673]	2429 [927]	874.9 [2620]	2372.9[2862]	916.4 [3435]	CG1116.4[3939]	1116.4[5896, iters=78]
CSTR50b50c1p5.dat	503.8 [982]	2420 [1629]	757.2 [4559]	2308.5[4998]	785.2 [5154]	CG879.1 [6318]	879.05[9116, iters=88]
CSTR50b50c1p6.dat	493.4 [990]	2445.3[1751]	678.9 [10330]	2305 [10999]	737.7 [19534]	CG840.1 [21234]	840.02[9946, iters=97]
CSTR50b50c1p7.dat	597.8 [570]	2488.4[901]	838.9 [3086]	2218.5[3828]	839.1 [4745]	CG994.8 [5533]	994.8[7194, iters=90]
CSTR50b50c2p01.dat	377.7 [814]	2489.3[1563]	633.6 [4520]	2287 [5635]	638.6 [6413]	CG672.3 [9355]	672.3[15436, iters=114]
CSTR50b50c2p02.dat	349.1 [1163]	2444.6[2031]	542.5 [5940]	2365.7[7118]	544.1 [8536]	CG593 [11136]	592.925[15110, iters=123]
CSTR50b50c2p03.dat	293.7 [1142]	2483.9[2037]	417 [5408]	2388.5[6005]	418.1 [7092]	CG448.2 [11697]	448.125[22206, iters=164]
CSTR50b50c2p4.dat	323.7 [1418]	2484.8[2337]	502.4 [5710]	2238.1[7217]	508.2 [8309]	CG534 [11452]	534[19176, iters=141]
CSTR50b50c2p5.dat	363.7 [1178]	2410.1[1978]	545.2 [5837]	2286.9[6402]	545.5 [7377]	CG578.7 [11966]	578.602[22276, iters=152]
CSTR50b50c2p6.dat	304.1 [1290]	2336.5[2211]	442.7 [9298]	2315.8[9815]	471.8 [14891]	CG509.1 [18327]	509.075[17963, iters=138]
CSTR50b50c2p7.dat	325.9 [1334]	2336.4[2240]	396.2 [8314]	2313 [8923]	537.7 [12581]	CG573.4 [18319]	573.359[25396, iters=176]
CSTR50b50c3p01.dat	166 [1086]	2359.7[2742]	171.1 [12882]	2359.7[13439]	171.1 [24638]	CG238.8 [34146]	238.713[38197, iters=200]
CSTR50b50c3p02.dat	140.4 [1503]	2473.1[3361]	166.9 [7461]	2399.5[8681]	167 [11387]	CG185.3 [20879]	185.2[31959, iters=155]
CSTR50b50c3p03.dat	158.4 [1473]	2337.4[3087]	158.7 [9361]	2305.9[10447]	200.2 [15462]	CG217.5 [21989]	217.5[27811, iters=154]
CSTR50b50c3p4.dat	165.1 [1732]	2437.1[3078]	185 [7288]	2410.7[8170]	185.2 [14806]	CG206.8 [21151]	206.8[29670, iters=166]
CSTR50b50c3p5.dat	158 [1559]	2483.9[3236]	190.6 [7815]	2389.7[9305]	190.8 [12786]	CG209.8 [20066]	209.8[28745, iters=158]
CSTR50b50c3p6.dat	161.2 [1506]	2407.2[3030]	163 [9230]	2397.1[10255]	163.1 [18019]	CG225.6 [24305]	225.55[27173, iters=152]
CSTR50b50c3p7.dat	163.2 [1518]	2454 [2886]	164.3 [14832]	2254.9[16209]	184.3 [30073]	CG201 [36378]	200.956[31718, iters=189]
CSTR50b50c4p01.dat	373.6 [695]	2500.1[1335]	543.8 [4138]	2224.6[5226]	544.3 [14353]	CG567.1 [16879]	567.09[17631, iters=146]
CSTR50b50c4p02.dat	346.6 [821]	2438 [1476]	499.5 [3784]	2191.7[4732]	500.6 [5726]	CG526.3 [9014]	526.25[19926, iters=160]
CSTR50b50c4p03.dat	537 [704]	2433.5[1146]	664.5 [3140]	2160.2[4029]	666.1 [4717]	CG700.1 [6932]	700.039[15888, iters=149]
CSTR50b50c4p4.dat	371.2 [1075]	2468.6[1715]	516.5 [4452]	2211.5[5328]	517.4 [5905]	CG548.7 [8445]	548.625[18451, iters=152]
CSTR50b50c4p5.dat	484.5 [704]	2500.1[1193]	622.9 [3402]	2237.2[4336]	624.8 [4707]	CG672.5 [7438]	672.47[16856, iters=151]
CSTR50b50c4p6.dat	483.3 [574]	2460.5[955]	610.6 [3324]	2242.2[4093]	611.2 [5112]	CG637.8 [6806]	637.725[13871, iters=139]
CSTR50b50c4p7.dat	465.8 [737]	2462.3[1235]	565.7 [3569]	2311.9[4319]	569.2 [5157]	CG588.5 [7615]	588.45[17058, iters=152]
CSTR50b50c5p01.dat	200 [866]	2381.3[2138]	210.1 [6162]	2157.3[7122]	210.2 [12898]	CG277.1 [17847]	277.037[28115, iters=200]
CSTR50b50c5p02.dat	204.6 [969]	2317.5[2088]	204.8 [10376]	2258.7[11215]	205.1 [20923]	CG294 [26178]	293.9[24195, iters=177]
CSTR50b50c5p03.dat	172.9 [1163]	2492 [2168]	217.9 [5276]	2290.1[6294]	218 [6659]	CG230.2 [12095]	230.2[26260, iters=185]
CSTR50b50c5p4.dat	202.7 [1266]	2419.4[2475]	202.7 [12507]	2256.8[13419]	204.4 [27555]	CG276.5 [32279]	276.494[23308, iters=170]
CSTR50b50c5p5.dat	179 [1164]	2419.1[2154]	209.4 [5454]	2356.2[6439]	210 [6642]	CG239 [11520]	238.938[27927, iters=201]
CSTR50b50c5p6.dat	201.3 [1177]	2440.7[2150]	270.5 [5780]	2433.5[6437]	271.6 [7239]	CG283.5 [12374]	283.452[26099, iters=184]
CSTR50b50c5p7.dat	191.6 [1002]	2400.9[1964]	191.7 [6390]	2284.1[7354]	245.6 [11721]	CG263.7 [16733]	263.7[22830, iters=168]