1 Second simulation (soundness)

1.1 Syntax

1.1.1 Index, indices and tables

%datatype *index*
%name *index* n α

$$\begin{array}{lll} n & ::= & 0 \\ & \mid & n+1 \end{array}$$

%datatype *vector*
%name *vector* $\mathcal{I}$

$$\begin{array}{lll} \mathcal{I} & ::= & [] \\ & \mid & n :: \mathcal{I} \end{array}$$

%datatype *table*
%name *table* $\mathcal{I}_\mu$

$$\begin{array}{lll} \mathcal{I}_\mu & ::= & [] \\ & \mid & \mathcal{I} :: \mathcal{I}_\mu \end{array}$$

1.1.2 Term

%datatype *term*
%name *term* t

$$\begin{array}{lll} t & ::= & n \\ & \mid & t_1\, t_2 \\ & \mid & \lambda t \\ & \mid & \textbf{get-context}\, t \\ & \mid & \textbf{set-context}\, \alpha\, t \end{array}$$

1.2 Subtraction

%judgment $n_1 \dot{-} n_2 = n_3$

$$n_1 \dot{-} 0 = n_1\, ^{[\mathrm{minus}_1]}$$
$$(n_1+1) \dot{-} (n_2+1) = n_3\, ^{[\mathrm{minus}_2]} \quad \textit{when} \quad n_1 \dot{-} n_2 = n_3$$

%mode $+n_1 \dot{-} +n_2 = -n_3$
%worlds $()$ $n_1 \dot{-} n_2 = n_3$
%terminates (n_1) $n_1 \dot{-} n_2 = n_3$
%unique $+n_1 \dot{-} +n_2 = -1n_3$

%lemma $\forall n \cdot n \dot{-} n = 0 \quad ^{[\mathrm{minus{\cdot}equals}]}$

Proof.

$$\frac{}{0\colon \textit{index} \cdot \quad 0 \dot{-} 0 = 0\, ^{[\mathrm{minus}_1]} \quad ^{[\mathrm{minus{\cdot}equals}]}}\,[\&1]$$

$$\frac{n\colon \textit{index} \cdot \quad \begin{array}{c}\mathcal{D}\\ n \dot{-} n = 0\end{array} \quad ^{[\mathrm{minus{\cdot}equals}]}}{n+1\colon \textit{index} \cdot \quad \dfrac{\begin{array}{c}\mathcal{D}\\ n \dot{-} n = 0\end{array}}{(n+1) \dot{-} (n+1) = 0}\, ^{[\mathrm{minus}_2]} \quad ^{[\mathrm{minus{\cdot}equals}]}}\,[\&2]$$

%mode $+n \cdot -\mathcal{D} \quad ^{[\mathrm{minus{\cdot}equals}]}$
%worlds $()$ $n \cdot \mathcal{D} \quad ^{[\mathrm{minus{\cdot}equals}]}$
%total n $n \cdot \mathcal{D} \quad ^{[\mathrm{minus{\cdot}equals}]}$

$\square$

1.2.1 Fetch (indices)

%judgment $\mathcal{I}(n_1) = n_2$

$$(n :: \mathcal{I})(0) = n\, ^{[\mathrm{fetch}^{\mathcal{I}}_1]}$$
$$(n :: \mathcal{I})(n_1+1) = n_2\, ^{[\mathrm{fetch}^{\mathcal{I}}_2]} \quad \textit{when} \quad \mathcal{I}(n_1) = n_2$$

%mode $+\mathcal{I}(+n_1) = -n_2$
%worlds $()$ $\mathcal{I}(n_1) = n_2$
%terminates n_1 $\mathcal{I}(n_1) = n_2$
%unique $+\mathcal{I}(+n_1) = -1n_2$

1.2.2 Fetch (table)

%judgment $\mathcal{I}_\mu(n) = \mathcal{I}$

$$(\mathcal{I} :: \mathcal{I}_\mu)(0) = \mathcal{I}\, ^{\left[\mathrm{fetch}^{\mathcal{I}_\mu}_1\right]}$$
$$(\mathcal{I}' :: \mathcal{I}_\mu)(\alpha+1) = \mathcal{I}\, ^{\left[\mathrm{fetch}^{\mathcal{I}_\mu}_2\right]} \quad \textit{when} \quad \mathcal{I}_\mu(\alpha) = \mathcal{I}$$

%mode $+\mathcal{I}_\mu(+\alpha) = -\mathcal{I}$
%worlds $()$ $\mathcal{I}_\mu(\alpha) = \mathcal{I}$
%terminates α $\mathcal{I}_\mu(\alpha) = \mathcal{I}$
%unique $+\mathcal{I}_\mu(+\alpha) = -1\mathcal{I}$

1.2.3 Compute

%judgment $n_1 \dot{-} \mathcal{I}(n_2) = n_3$

$$n \dot{-} \mathcal{I}(l) = g\, ^{[\mathrm{compute}_1]} \quad \textit{when} \quad \mathcal{I}(l) = k, \quad n \dot{-} k = g$$

%mode $+n_1 \dot{-} +\mathcal{I}(+n_2) = -n_3$
%worlds $()$ $n_1 \dot{-} \mathcal{I}(n_2) = n_3$
%terminates $\{\}$ $n_1 \dot{-} \mathcal{I}(n_2) = n_3$
%unique $+n_1 \dot{-} +\mathcal{I}(+n_2) = -1n_3$

1.2.4 Closure, environment and stack

%datatype *clos* **%name** *clos* c
%datatype *l-env* **%name** *l-env* $\mathcal{L}$
%datatype *l-table* **%name** *l-table* $\mathcal{L}_\mu$
%datatype *k-env* **%name** *k-env* $\mathcal{E}_\mu$
%datatype *stack* **%name** *stack* $\mathcal{S}$

$$\begin{array}{lll} c & ::= & (t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu) \end{array}$$

$$\begin{array}{lll} \mathcal{L} & ::= & () \\ & \mid & (c; \mathcal{L}) \end{array}$$

$$\begin{array}{lll} \mathcal{L}_\mu & ::= & () \\ & \mid & \mathcal{L}; \mathcal{L}_\mu \end{array}$$

$$\begin{array}{lll} \mathcal{E}_\mu & ::= & () \\ & \mid & (\mathcal{S}; \mathcal{E}_\mu) \end{array}$$

$$\begin{array}{lll} \mathcal{S} & ::= & [] \\ & \mid & c :: \mathcal{S} \end{array}$$

%datatype *state*
%name *state* σ

$$\begin{array}{lll} \sigma & ::= & \langle t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S} \rangle \end{array}$$

1.3 Judgments

1.3.1 Fetch a local closure

%judgment $\mathcal{L}(n) = c$

$$(c; \mathcal{L})(0) = c\, ^{[\mathrm{fetch}_1]}$$
$$(c'; \mathcal{L})(n+1) = c\, ^{[\mathrm{fetch}_2]} \quad \textit{when} \quad \mathcal{L}(n) = c$$

%mode $+\mathcal{L}(+n) = -c$
%worlds $()$ $\mathcal{L}(n) = c$
%terminates $\mathcal{L}$ $\mathcal{L}(n) = c$
%unique $+\mathcal{L}(+n) = -1c$

1.3.2 Fetch a local environment

%judgment $\mathcal{L}_\mu(n) = \mathcal{L}$

$$(\mathcal{L}; \mathcal{L}_\mu)(0) = \mathcal{L}\, ^{[\mathrm{l{\cdot}fetch}_1]}$$
$$(\mathcal{L}'; \mathcal{L}_\mu)(n+1) = \mathcal{L}\, ^{[\mathrm{l{\cdot}fetch}_2]} \quad \textit{when} \quad \mathcal{L}_\mu(n) = \mathcal{L}$$

%mode $+\mathcal{L}_\mu(+n) = -\mathcal{L}$
%worlds $()$ $\mathcal{L}_\mu(n) = \mathcal{L}$
%terminates $\mathcal{L}_\mu$ $\mathcal{L}_\mu(n) = \mathcal{L}$
%unique $+\mathcal{L}_\mu(+n) = -1\mathcal{L}$

1.3.3 Fetch a stack

%judgment $\mathcal{E}_\mu(n) = \mathcal{S}$

$$(\mathcal{S}; \mathcal{E}_\mu)(0) = \mathcal{S}\, ^{[\mathrm{fetch}^{\mu}_1]}$$
$$(\mathcal{S}'; \mathcal{E}_\mu)(n+1) = \mathcal{S}\, ^{[\mathrm{fetch}^{\mu}_2]} \quad \textit{when} \quad \mathcal{E}_\mu(n) = \mathcal{S}$$

%mode $+\mathcal{E}_\mu(+n) = -\mathcal{S}$
%worlds $()$ $\mathcal{E}_\mu(n) = \mathcal{S}$
%terminates $\mathcal{E}_\mu$ $\mathcal{E}_\mu(n) = \mathcal{S}$
%unique $+\mathcal{E}_\mu(+n) = -1\mathcal{S}$

1.3.4 Evaluation rules

%judgment $\sigma_1 \rightsquigarrow \sigma_2$

$$\langle k, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S} \rangle \rightsquigarrow \langle t, \mathcal{L}', \mathcal{L}'_\mu, \mathcal{E}'_\mu, \mathcal{S} \rangle\, ^{[\mathrm{k{\cdot}var}]} \quad \textit{when} \quad \mathcal{L}(k) = (t, \mathcal{L}', \mathcal{L}'_\mu, \mathcal{E}'_\mu)$$
$$\langle (t\, u), \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S} \rangle \rightsquigarrow \langle t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, (u, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu) :: \mathcal{S} \rangle\, ^{[\mathrm{k{\cdot}app}]}$$
$$\langle \lambda t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, c :: \mathcal{S} \rangle \rightsquigarrow \langle t, (c; \mathcal{L}), \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S} \rangle\, ^{[\mathrm{k{\cdot}abs}]}$$
$$\langle \textbf{get-context}\, t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S} \rangle \rightsquigarrow \langle t, \mathcal{L}, (\mathcal{L}; \mathcal{L}_\mu), (\mathcal{S}; \mathcal{E}_\mu), \mathcal{S} \rangle\, ^{[\mathrm{k{\cdot}catch}]}$$
$$\langle \textbf{set-context}\, \alpha\, t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S} \rangle \rightsquigarrow \langle t, \mathcal{L}', \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}' \rangle\, ^{[\mathrm{k{\cdot}throw}]} \quad \textit{when} \quad \mathcal{L}_\mu(\alpha) = \mathcal{L}', \quad \mathcal{E}_\mu(\alpha) = \mathcal{S}'$$

%mode $+\sigma_1 \rightsquigarrow -\sigma_2$
%worlds $()$ $\sigma_1 \rightsquigarrow \sigma_2$
%unique $+\sigma_1 \rightsquigarrow -1\sigma_2$

1.4 Abstract machine for safe $\boldsymbol{\lambda}_{\mathbf{ct}}$-terms

1.4.1 Syntax

%datatype *clos*
%datatype *c-env*
%datatype *k-env*
%datatype *stack*
%name *clos* $\tilde{c}$
%name *c-env* $\tilde{\mathcal{E}}$
%name *k-env* $\tilde{\mathcal{E}}_\mu$
%name *stack* $\tilde{\mathcal{S}}$

$$\begin{array}{lll} \tilde{c} & ::= & \big(t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu\big) \end{array}$$

$$\begin{array}{lll} \tilde{\mathcal{E}} & ::= & () \\ & \mid & (\tilde{c}; \tilde{\mathcal{E}}) \end{array}$$

$$\begin{array}{lll} \tilde{\mathcal{E}}_\mu & ::= & () \\ & \mid & (\tilde{\mathcal{S}}; \tilde{\mathcal{E}}_\mu) \end{array}$$

$$\begin{array}{lll} \tilde{\mathcal{S}} & ::= & [] \\ & \mid & \tilde{c} :: \tilde{\mathcal{S}} \end{array}$$

%datatype *state*
%name *state* $\tilde{\sigma}$

$$\begin{array}{lll} \tilde{\sigma} & ::= & \langle t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}} \rangle \end{array}$$

1.4.2 Fetch a closure

%judgment $\tilde{\mathcal{E}}(n) = \tilde{c}$

$$(\tilde{c}; \tilde{\mathcal{E}})(0) = \tilde{c}\, ^{[\mathrm{i{\cdot}fetch}_1]}$$
$$(\tilde{c}'; \tilde{\mathcal{E}})(n+1) = \tilde{c}\, ^{[\mathrm{i{\cdot}fetch}_2]} \quad \textit{when} \quad \tilde{\mathcal{E}}(n) = \tilde{c}$$

%mode $+\tilde{\mathcal{E}}(+n)=-\tilde{c}$
%worlds $()\quad \tilde{\mathcal{E}}(n)=\tilde{c}$
%terminates $\tilde{\mathcal{E}}\quad \tilde{\mathcal{E}}(n)=\tilde{c}$
%unique $+\tilde{\mathcal{E}}(+n)=-1\tilde{c}$

1.4.3 Compute

%judgment $\tilde{\mathcal{E}}(n_1 \mathbin{\dot{-}} n_2)=\tilde{c}$

$$\tilde{\mathcal{E}}(n \mathbin{\dot{-}} k)=\tilde{c}^{\,[\text{i}\cdot\text{compute}_1]} \quad \textit{when} \quad n \mathbin{\dot{-}} k = g, \quad \tilde{\mathcal{E}}(g)=\tilde{c}$$

%mode $+\tilde{\mathcal{E}}(+n \mathbin{\dot{-}} +k)=-\tilde{c}$
%worlds $()\quad \tilde{\mathcal{E}}(n \mathbin{\dot{-}} k)=\tilde{c}$
%terminates $\{\}\quad \tilde{\mathcal{E}}(n \mathbin{\dot{-}} k)=\tilde{c}$
%unique $+\tilde{\mathcal{E}}(+n \mathbin{\dot{-}} +k)=-1\tilde{c}$

1.4.4 Fetch a stack

%judgment $\tilde{\mathcal{E}}_\mu(n)=\tilde{\mathcal{S}}$

$$(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu)(0)=\tilde{\mathcal{S}}^{\,[\text{i}\cdot\text{fetch}^\mu_1]}$$
$$(\tilde{\mathcal{S}}';\tilde{\mathcal{E}}_\mu)(n+1)=\tilde{\mathcal{S}}^{\,[\text{i}\cdot\text{fetch}^\mu_2]} \quad \textit{when} \quad \tilde{\mathcal{E}}_\mu(n)=\tilde{\mathcal{S}}$$

%mode $+\tilde{\mathcal{E}}_\mu(+n)=-\tilde{\mathcal{S}}$
%worlds $()\quad \tilde{\mathcal{E}}_\mu(n)=\tilde{\mathcal{S}}$
%terminates $\tilde{\mathcal{E}}_\mu\quad \tilde{\mathcal{E}}_\mu(n)=\tilde{\mathcal{S}}$
%unique $+\tilde{\mathcal{E}}_\mu(+n)=-1\tilde{\mathcal{S}}$

2 Evaluation rules

%judgment $\tilde{\sigma}_1 \rightsquigarrow \tilde{\sigma}_2$

$$\langle l,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t,n',\mathcal{I}',\mathcal{I}'_\mu,\tilde{\mathcal{E}}',\tilde{\mathcal{E}}'_\mu,\tilde{\mathcal{S}}\rangle^{\,[\text{i}\cdot\text{var}]}$$
$$\textit{when} \quad n \mathbin{\dot{-}} \mathcal{I}(l)=g, \quad \tilde{\mathcal{E}}(g)=\big(t,n',\mathcal{I}',\mathcal{I}'_\mu,\tilde{\mathcal{E}}',\tilde{\mathcal{E}}'_\mu\big)$$
$$\langle (t\,u),n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\big(u,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu\big)::\tilde{\mathcal{S}}\rangle^{\,[\text{i}\cdot\text{app}]}$$
$$\langle \lambda t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{c}::\tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t,n+1,(n+1::\mathcal{I}),\mathcal{I}_\mu,(\tilde{c};\tilde{\mathcal{E}}),\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle^{\,[\text{i}\cdot\text{abs}]}$$
$$\langle \textbf{get-context}\,t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t,n,\mathcal{I},(\mathcal{I}::\mathcal{I}_\mu),\tilde{\mathcal{E}},(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu),\tilde{\mathcal{S}}\rangle^{\,[\text{i}\cdot\text{catch}]}$$
$$\langle \textbf{set-context}\,\alpha\,t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t,n,\mathcal{I}',\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}'\rangle^{\,[\text{i}\cdot\text{throw}]}$$
$$\textit{when} \quad \mathcal{I}_\mu(\alpha)=\mathcal{I}', \quad \tilde{\mathcal{E}}_\mu(\alpha)=\tilde{\mathcal{S}}'$$

%mode $+\sigma_1 \rightsquigarrow -\sigma_2$
%worlds $()\quad \sigma_1 \rightsquigarrow \sigma_2$
%unique $+\sigma_1 \rightsquigarrow -1\sigma_2$

3 Translation

%judgment $\tilde{c}^\diamond = c$
%judgment $\tilde{\mathcal{S}}^\diamond = \mathcal{S}$
%judgment $\tilde{\mathcal{E}}_\mu{}^\diamond = \mathcal{E}_\mu$
%judgment $\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}$
%judgment $\textit{map}\ \big(\textit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ \mathcal{I}_\mu = \mathcal{L}_\mu$

$$\big(t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu\big)^\diamond = (t,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu)^{\,[\text{clos}^\diamond]} \quad \textit{when} \quad \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I}=\mathcal{L}, \quad \textit{map}\ \big(\textit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ \mathcal{I}_\mu=\mathcal{L}_\mu, \quad \tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu$$

$$[]^\diamond = []^{\,[\text{stack}^\diamond_1]}$$
$$\big(\tilde{c}::\tilde{\mathcal{S}}\big)^\diamond = c::\mathcal{S}^{\,[\text{stack}^\diamond_2]} \quad \textit{when} \quad \tilde{c}^\diamond=c, \quad \tilde{\mathcal{S}}^\diamond=\mathcal{S}$$

$$()^\diamond = ()^{\,[\text{k}\cdot\text{env}^\diamond_1]}$$
$$(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu)^\diamond = (\mathcal{S};\mathcal{E}_\mu)^{\,[\text{k}\cdot\text{env}^\diamond_2]} \quad \textit{when} \quad \tilde{\mathcal{S}}^\diamond=\mathcal{S}, \quad \tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu$$

$$\textit{flatten}\ n\ \tilde{\mathcal{E}}\ [] = ()^{\,[\text{flatten}_1]}$$
$$\textit{flatten}\ n\ \tilde{\mathcal{E}}\ (k::\mathcal{I}) = (c;\mathcal{L})^{\,[\text{flatten}_2]} \quad \textit{when} \quad \tilde{\mathcal{E}}(n \mathbin{\dot{-}} k)=\tilde{c}, \quad \tilde{c}^\diamond=c, \quad \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I}=\mathcal{L}$$

$$\textit{map}\ \big(\textit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ [] = ()^{\,[\text{map}_1]}$$
$$\textit{map}\ \big(\textit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ (\mathcal{I}::\mathcal{I}_\mu) = \mathcal{L};\mathcal{L}_\mu{}^{\,[\text{map}_2]} \quad \textit{when} \quad \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I}=\mathcal{L}, \quad \textit{map}\ \big(\textit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ \mathcal{I}_\mu=\mathcal{L}_\mu$$

%mode
$+\tilde{c}^\diamond = -c$
$+\tilde{\mathcal{S}}^\diamond = -\mathcal{S}$
$+\tilde{\mathcal{E}}_\mu{}^\diamond = -\mathcal{E}_\mu$
$\textit{flatten}\ +n\ +\tilde{\mathcal{E}}\ +\mathcal{I} = -\mathcal{L}$
$\textit{map}\ \big(\textit{flatten}\ +n\ +\tilde{\mathcal{E}}\big)\ +\mathcal{I}_\mu = -\mathcal{L}_\mu$

%worlds $()$
$\tilde{c}^\diamond = c$
$\tilde{\mathcal{S}}^\diamond = \mathcal{S}$
$\tilde{\mathcal{E}}_\mu{}^\diamond = \mathcal{E}_\mu$
$\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}$
$\textit{map}\ \big(\textit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ \mathcal{I}_\mu = \mathcal{L}_\mu$

Remark. To do.

%terminates $\big(\tilde{c}\ \tilde{\mathcal{S}}\ \tilde{\mathcal{E}}_\mu\ \mathcal{I}\ \mathcal{I}_\mu\big)$
$\tilde{c}^\diamond = c$
$\tilde{\mathcal{S}}^\diamond = \mathcal{S}$
$\tilde{\mathcal{E}}_\mu{}^\diamond = \mathcal{E}_\mu$
$\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}$
$\textit{map}\ \big(\textit{flatten}\ n'\ \tilde{\mathcal{E}}'\big)\ \mathcal{I}_\mu = \mathcal{L}_\mu$

%unique
$+\tilde{c}^\diamond = -1c$
$+\tilde{\mathcal{S}}^\diamond = -1\mathcal{S}$
$+\tilde{\mathcal{E}}_\mu{}^\diamond = -1\mathcal{E}_\mu$
$\textit{flatten}\ +n\ +\tilde{\mathcal{E}}\ +\mathcal{I} = -1\mathcal{L}$
$\textit{map}\ \big(\textit{flatten}\ +n\ +\tilde{\mathcal{E}}\big)\ +\mathcal{I}_\mu = -1\mathcal{L}_\mu$

%judgment $\tilde{\sigma}^\diamond = \sigma$
$$\langle t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle^\diamond = \langle t',\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu,\mathcal{S}\rangle^{\,[\text{state}^\diamond]} \quad \textit{when} \quad \big(t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu\big)^\diamond = (t',\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu), \quad \tilde{\mathcal{S}}^\diamond = \mathcal{S}$$

%mode $+\tilde{\sigma}^\diamond = -\sigma$
%worlds $()\quad \tilde{\sigma}^\diamond = \sigma$
%unique $+\tilde{\sigma}^\diamond = -1\sigma$

4 Soundness

%lemma $n \mathbin{\dot{-}} k = g \quad \Rightarrow \quad (n+1) \mathbin{\dot{-}} k = g+1 \quad ^{[\text{minus}\cdot\text{succ}]}$

Proof.

$$\dfrac{}{n \mathbin{\dot{-}} 0 = n^{\,[\text{minus}_1]} \quad \Rightarrow \quad (n+1) \mathbin{\dot{-}} 0 = (n+1)^{\,[\text{minus}_1]} \quad ^{[\text{minus}\cdot\text{succ}]}}\,[\&1]$$

$$\dfrac{\overset{\mathcal{D}_1}{n \mathbin{\dot{-}} k = g} \quad \Rightarrow \quad \overset{\mathcal{D}_2}{(n+1) \mathbin{\dot{-}} k = g+1} \quad ^{[\text{minus}\cdot\text{succ}]}}{\dfrac{\overset{\mathcal{D}_1}{n \mathbin{\dot{-}} k = g}}{(n+1) \mathbin{\dot{-}} (k+1) = g}\,[\text{minus}_2] \quad \Rightarrow \quad \dfrac{\overset{\mathcal{D}_2}{(n+1) \mathbin{\dot{-}} k = g+1}}{(n+1+1) \mathbin{\dot{-}} (k+1) = g+1}\,[\text{minus}_2] \quad ^{[\text{minus}\cdot\text{succ}]}}\,[\&1]$$

%mode $+\mathcal{D}_1 \quad \Rightarrow \quad -\mathcal{D}_2 \quad ^{[\text{minus}\cdot\text{succ}]}$
%worlds $()\quad \mathcal{D}_1 \quad \Rightarrow \quad \mathcal{D}_2 \quad ^{[\text{minus}\cdot\text{succ}]}$
%total $(\mathcal{D}_1)\quad \mathcal{D}_1 \quad \Rightarrow \quad \mathcal{D}_2 \quad ^{[\text{minus}\cdot\text{succ}]}$

□

%lemma $\forall \tilde{c}' \cdot \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L} \quad \Rightarrow \quad \textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\ \mathcal{I} = \mathcal{L} \quad ^{[\text{weaken}\cdot\text{flatten}]}$

Proof.

$$\dfrac{}{\tilde{c}' : \textit{clos} \cdot \ \textit{flatten}\ n\ \tilde{\mathcal{E}}\ [] = ()^{\,[\text{flatten}_1]} \quad \Rightarrow \quad \textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\ [] = ()^{\,[\text{flatten}_1]} \quad ^{[\text{weaken}\cdot\text{flatten}]}}\,[\&1]$$

$$\dfrac{\begin{array}{c}\tilde{c}' : \textit{clos} \cdot \ \overset{\mathcal{D}_{23}}{\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}} \quad \Rightarrow \quad \overset{\mathcal{D}_3}{\textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\ \mathcal{I} = \mathcal{L}} \quad ^{[\text{weaken}\cdot\text{flatten}]} \\ \overset{\mathcal{D}_{211}}{n \mathbin{\dot{-}} k = g} \quad \Rightarrow \quad \overset{\mathcal{D}_4}{(n+1) \mathbin{\dot{-}} k = g+1} \quad ^{[\text{minus}\cdot\text{succ}]}\end{array}}{\tilde{c}' : \textit{clos} \cdot \ \dfrac{\dfrac{\overset{\mathcal{D}_{211}}{n \mathbin{\dot{-}} k = g} \quad \overset{\mathcal{D}_{212}}{\tilde{\mathcal{E}}(g) = \tilde{c}}}{\tilde{\mathcal{E}}(n \mathbin{\dot{-}} k) = \tilde{c}}\,[\text{i}\cdot\text{compute}_1] \quad \overset{\mathcal{D}_{22}}{\tilde{c}^\diamond = c} \quad \overset{\mathcal{D}_{23}}{\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}}}{\textit{flatten}\ n\ \tilde{\mathcal{E}}\ (k::\mathcal{I}) = (c;\mathcal{L})}\,[\text{flatten}_2] \quad \Rightarrow \quad \dfrac{\dfrac{\overset{\mathcal{D}_4}{(n+1) \mathbin{\dot{-}} k = g+1} \quad \overset{\mathcal{D}_{212}}{\dfrac{\tilde{\mathcal{E}}(g) = \tilde{c}}{(\tilde{c}';\tilde{\mathcal{E}})(g+1) = \tilde{c}}}\,[\text{i}\cdot\text{fetch}_2]}{(\tilde{c}';\tilde{\mathcal{E}})(n+1 \mathbin{\dot{-}} k) = \tilde{c}}\,[\text{i}\cdot\text{compute}_1] \quad \overset{\mathcal{D}_{22}}{\tilde{c}^\diamond = c} \quad \overset{\mathcal{D}_3}{\textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\ \mathcal{I} = \mathcal{L}}}{\textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\ (k::\mathcal{I}) = (c;\mathcal{L})}\,[\text{flatten}_2] \quad ^{[\text{weaken}\cdot\text{flatten}]}}\,[\&2]$$

%mode $+\tilde{c}' \cdot +\mathcal{D}_1 \quad \Rightarrow \quad -\mathcal{D}_2 \quad ^{[\text{weaken}\cdot\text{flatten}]}$
%worlds $()\quad \tilde{c}' \cdot \mathcal{D}_1 \quad \Rightarrow \quad \mathcal{D}_2 \quad ^{[\text{weaken}\cdot\text{flatten}]}$
%total $(\mathcal{D}_1)\quad \tilde{c}' \cdot \mathcal{D}_1 \quad \Rightarrow \quad \mathcal{D}_2 \quad ^{[\text{weaken}\cdot\text{flatten}]}$

□

%lemma $\forall \tilde{c}' \cdot \textit{map}\ \big(\textit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ \mathcal{I}_\mu = \mathcal{L}_\mu \quad \Rightarrow \quad \textit{map}\ \big(\textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\big)\ \mathcal{I}_\mu = \mathcal{L}_\mu \quad ^{[\text{weaken}\cdot\text{map}]}$

Proof.

$$\dfrac{}{\tilde{c}' : \textit{clos} \cdot \ \textit{map}\ \big(\textit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ [] = ()^{\,[\text{map}_1]} \quad \Rightarrow \quad \textit{map}\ \big(\textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\big)\ [] = ()^{\,[\text{map}_1]} \quad ^{[\text{weaken}\cdot\text{map}]}}\,[\&1]$$

$$\dfrac{\begin{array}{c}\tilde{c}' : \textit{clos} \cdot \ \overset{\mathcal{D}_{12}}{\textit{map}\ \big(\textit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ \mathcal{I}_\mu = \mathcal{L}_\mu} \quad \Rightarrow \quad \overset{\mathcal{D}_2}{\textit{map}\ \big(\textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\big)\ \mathcal{I}_\mu = \mathcal{L}_\mu} \quad ^{[\text{weaken}\cdot\text{map}]} \\ \tilde{c}' : \textit{clos} \cdot \ \overset{\mathcal{D}_{11}}{\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}} \quad \Rightarrow \quad \overset{\mathcal{D}_3}{\textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\ \mathcal{I} = \mathcal{L}} \quad ^{[\text{weaken}\cdot\text{flatten}]}\end{array}}{\tilde{c}' : \textit{clos} \cdot \ \dfrac{\overset{\mathcal{D}_{11}}{\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}} \quad \overset{\mathcal{D}_{12}}{\textit{map}\ \big(\textit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ \mathcal{I}_\mu = \mathcal{L}_\mu}}{\textit{map}\ \big(\textit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ (\mathcal{I}::\mathcal{I}_\mu) = (\mathcal{L};\mathcal{L}_\mu)}\,[\text{map}_2] \quad \Rightarrow \quad \dfrac{\overset{\mathcal{D}_3}{\textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\ \mathcal{I} = \mathcal{L}} \quad \overset{\mathcal{D}_2}{\textit{map}\ \big(\textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\big)\ \mathcal{I}_\mu = \mathcal{L}_\mu}}{\textit{map}\ \big(\textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\big)\ (\mathcal{I}::\mathcal{I}_\mu) = (\mathcal{L};\mathcal{L}_\mu)}\,[\text{map}_2] \quad ^{[\text{weaken}\cdot\text{map}]}}\,[\&2]$$

%mode $+\tilde{c}' \cdot +\mathcal{D}_1 \quad \Rightarrow \quad -\mathcal{D}_2 \quad ^{[\text{weaken}\cdot\text{map}]}$
%worlds $()\quad \tilde{c}' \cdot \mathcal{D}_1 \quad \Rightarrow \quad \mathcal{D}_2 \quad ^{[\text{weaken}\cdot\text{map}]}$
%total $(\mathcal{D}_1)\quad \tilde{c}' \cdot \mathcal{D}_1 \quad \Rightarrow \quad \mathcal{D}_2 \quad ^{[\text{weaken}\cdot\text{map}]}$

□

%lemma $\textit{map}\ \big(\textit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ \mathcal{I}_\mu = \mathcal{L}_\mu \quad \wedge \quad \mathcal{I}_\mu(\alpha) = \mathcal{I}' \quad \Rightarrow \quad \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I}' = \mathcal{L}' \quad \wedge \quad \mathcal{L}_\mu(\alpha) = \mathcal{L}' \quad ^{[\text{map}\cdot\text{sound}]}$

Proof.

$$\dfrac{}{\dfrac{\overset{\mathcal{D}_{11}}{\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}} \quad \overset{\vdots}{\textit{map}\ \big(\textit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ \mathcal{I}_\mu = \mathcal{L}_\mu}}{\textit{map}\ \big(\textit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ (\mathcal{I}::\mathcal{I}_\mu) = (\mathcal{L};\mathcal{L}_\mu)}\,[\text{map}_2] \quad \wedge \quad (\mathcal{I}::\mathcal{I}_\mu)(0) = \mathcal{I}^{\,\left[\text{fetch}^{\mathcal{I}_\mu}_1\right]} \quad \Rightarrow \quad \overset{\mathcal{D}_{11}}{\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}} \quad \wedge \quad (\mathcal{L};\mathcal{L}_\mu)(0) = \mathcal{L}^{\,[\text{l}\cdot\text{fetch}_1]} \quad ^{[\text{map}\cdot\text{sound}]}}\,[\&1]$$

$$\dfrac{\overset{\mathcal{D}_{12}}{\textit{map}\ \big(\textit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ \mathcal{I}_\mu = \mathcal{L}_\mu} \quad \wedge \quad \overset{\mathcal{D}_2}{\mathcal{I}_\mu(\alpha) = \mathcal{I}'} \quad \Rightarrow \quad \overset{\mathcal{D}_3}{\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I}' = \mathcal{L}'} \quad \wedge \quad \overset{\mathcal{D}_4}{\mathcal{L}_\mu(\alpha) = \mathcal{L}'} \quad ^{[\text{map}\cdot\text{sound}]}}{\dfrac{\overset{\vdots}{\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}} \quad \overset{\mathcal{D}_{12}}{\textit{map}\ \big(\textit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ \mathcal{I}_\mu = \mathcal{L}_\mu}}{\textit{map}\ \big(\textit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ (\mathcal{I}::\mathcal{I}_\mu) = (\mathcal{L};\mathcal{L}_\mu)}\,[\text{map}_2] \quad \wedge \quad \overset{\mathcal{D}_2}{\dfrac{\mathcal{I}_\mu(\alpha) = \mathcal{I}'}{(\mathcal{I}::\mathcal{I}_\mu)(\alpha+1) = \mathcal{I}'}}\,\left[\text{fetch}^{\mathcal{I}_\mu}_2\right] \quad \Rightarrow \quad \overset{\mathcal{D}_3}{\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I}' = \mathcal{L}'} \quad \wedge \quad \overset{\mathcal{D}_4}{\dfrac{\mathcal{L}_\mu(\alpha) = \mathcal{L}'}{(\mathcal{L};\mathcal{L}_\mu)(\alpha+1) = \mathcal{L}'}}\,[\text{l}\cdot\text{fetch}_2] \quad ^{[\text{map}\cdot\text{sound}]}}\,[\&2]$$

%mode $+\mathcal{D}_1 \quad \wedge \quad +\mathcal{D}_2 \quad \Rightarrow \quad -\mathcal{D}_3 \quad \wedge \quad -\mathcal{D}_4 \quad ^{[\text{map}\cdot\text{sound}]}$
%worlds $()\quad \mathcal{D}_1 \quad \wedge \quad \mathcal{D}_2 \quad \Rightarrow \quad \mathcal{D}_3 \quad \wedge \quad \mathcal{D}_4 \quad ^{[\text{map}\cdot\text{sound}]}$
%total $(\mathcal{D}_2)\quad \mathcal{D}_1 \quad \wedge \quad \mathcal{D}_2 \quad \Rightarrow \quad \mathcal{D}_3 \quad \wedge \quad \mathcal{D}_4 \quad ^{[\text{map}\cdot\text{sound}]}$

□

%lemma $\mathcal{I}(l) = k \quad \wedge \quad \tilde{\mathcal{E}}(n \mathbin{\dot{-}} k) = \tilde{c} \quad \wedge \quad \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L} \quad \Rightarrow \quad \tilde{c}^\diamond = c \quad \wedge \quad \mathcal{L}(l) = c \quad ^{[\text{fetch}\cdot\text{sound}]}$

Proof.

$$\dfrac{}{(k::\mathcal{I})(0) = k^{\,[\text{fetch}^{\mathcal{I}}_1]} \quad \wedge \quad \overset{\mathcal{D}_2}{\tilde{\mathcal{E}}(n \mathbin{\dot{-}} k) = \tilde{c}} \quad \wedge \quad \dfrac{\overset{\mathcal{D}_{31}}{\tilde{\mathcal{E}}(n \mathbin{\dot{-}} k) = \tilde{c}} \quad \overset{\mathcal{D}_{32}}{\tilde{c}^\diamond = c} \quad \overset{\vdots}{\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}}}{\textit{flatten}\ n\ \tilde{\mathcal{E}}\ (k::\mathcal{I}) = (c;\mathcal{L})}\,[\text{flatten}_2] \quad \Rightarrow \quad \overset{\mathcal{D}_{32}}{\tilde{c}^\diamond = c} \quad \wedge \quad (c;\mathcal{L})(0) = c^{\,[\text{fetch}_1]} \quad ^{[\text{fetch}\cdot\text{sound}]}}\,[\&1]$$

$$\dfrac{\overset{\mathcal{D}_{11}}{\mathcal{I}(l) = k} \quad \wedge \quad \overset{\mathcal{D}_2}{\tilde{\mathcal{E}}(n \mathbin{\dot{-}} k) = \tilde{c}} \quad \wedge \quad \overset{\mathcal{D}_{33}}{\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}} \quad \Rightarrow \quad \overset{\mathcal{D}_4}{\tilde{c}^\diamond = c} \quad \wedge \quad \overset{\mathcal{D}_5}{\mathcal{L}(l) = c} \quad ^{[\text{fetch}\cdot\text{sound}]}}{\overset{\mathcal{D}_{11}}{\dfrac{\mathcal{I}(l) = k}{(k'::\mathcal{I})(l+1) = k}}\,[\text{fetch}^{\mathcal{I}}_2] \quad \wedge \quad \overset{\mathcal{D}_2}{\tilde{\mathcal{E}}(n \mathbin{\dot{-}} k) = \tilde{c}} \quad \wedge \quad \dfrac{\overset{\mathcal{D}_{31}}{\tilde{\mathcal{E}}(n \mathbin{\dot{-}} k') = \tilde{c}'} \quad \overset{\mathcal{D}_{32}}{\tilde{c}'^\diamond = c'} \quad \overset{\mathcal{D}_{33}}{\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}}}{\textit{flatten}\ n\ \tilde{\mathcal{E}}\ (k'::\mathcal{I}) = (c';\mathcal{L})}\,[\text{flatten}_2] \quad \Rightarrow \quad \overset{\mathcal{D}_4}{\tilde{c}^\diamond = c} \quad \wedge \quad \overset{\mathcal{D}_5}{\dfrac{\mathcal{L}(l) = c}{(c';\mathcal{L})(l+1) = c}}\,[\text{fetch}_2] \quad ^{[\text{fetch}\cdot\text{sound}]}}\,[\&2]$$

%mode $+\mathcal{D}_1 \;\wedge\; +\mathcal{D}_2 \;\wedge\; +\mathcal{D}_3 \;\Rightarrow\; -\mathcal{D}_4 \;\wedge\; -\mathcal{D}_5$ [fetch·sound]
%worlds $()\;\mathcal{D}_1 \wedge \mathcal{D}_2 \wedge \mathcal{D}_3 \Rightarrow \mathcal{D}_4 \wedge \mathcal{D}_5$ [fetch·sound]
%total $(\mathcal{D}_1)\;\mathcal{D}_1 \wedge \mathcal{D}_2 \wedge \mathcal{D}_3 \Rightarrow \mathcal{D}_4 \wedge \mathcal{D}_5$ [fetch·sound] □

%lemma $\tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu \;\wedge\; \tilde{\mathcal{E}}_\mu(\alpha)=\tilde{\mathcal{S}} \;\Rightarrow\; \tilde{\mathcal{S}}^\diamond=\mathcal{S} \;\wedge\; \mathcal{E}_\mu(\alpha)=\mathcal{S}$ [fetch$^\mu$·sound]

Proof.

$$\dfrac{}{\dfrac{\overset{\mathcal{D}_{11}}{\tilde{\mathcal{S}}^\diamond=\mathcal{S}}\quad \overset{\vdots}{\tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu}}{(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu)^\diamond=(\mathcal{S};\mathcal{E}_\mu)}[\text{k·env}^\diamond_2] \;\wedge\; (\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu)(0)=\tilde{\mathcal{S}}\,[\text{i·fetch}^\mu_1] \;\Rightarrow\; \overset{\mathcal{D}_{11}}{\tilde{\mathcal{S}}^\diamond=\mathcal{S}} \;\wedge\; (\mathcal{S};\mathcal{E}_\mu)(0)=\mathcal{S}\,[\text{fetch}^\mu_1]\quad[\text{fetch}^\mu\text{·sound}]}[\&1]$$

$$\dfrac{\overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu} \wedge \overset{\mathcal{D}_2}{\tilde{\mathcal{E}}_\mu(\alpha)=\tilde{\mathcal{S}}} \Rightarrow \overset{\mathcal{D}_3}{\tilde{\mathcal{S}}^\diamond=\mathcal{S}} \wedge \overset{\mathcal{D}_4}{\mathcal{E}_\mu(\alpha)=\mathcal{S}}\quad[\text{fetch}^\mu\text{·sound}]}{\dfrac{\overset{\vdots}{\tilde{\mathcal{S}}'^\diamond=\mathcal{S}'}\quad \overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu}}{(\tilde{\mathcal{S}}';\tilde{\mathcal{E}}_\mu)^\diamond=(\tilde{\mathcal{S}}';\mathcal{E}_\mu)}[\text{k·env}^\diamond_2] \;\wedge\; \dfrac{\overset{\mathcal{D}_2}{\tilde{\mathcal{E}}_\mu(\alpha)=\tilde{\mathcal{S}}}}{(\tilde{\mathcal{S}}';\tilde{\mathcal{E}}_\mu)(\alpha+1)=\tilde{\mathcal{S}}}[\text{i·fetch}^\mu_2] \;\Rightarrow\; \overset{\mathcal{D}_3}{\tilde{\mathcal{S}}^\diamond=\mathcal{S}} \;\wedge\; \dfrac{\overset{\mathcal{D}_4}{\mathcal{E}_\mu(\alpha)=\mathcal{S}}}{(\mathcal{S}';\mathcal{E}_\mu)(n+1)=\mathcal{S}}[\text{fetch}^\mu_2]\quad[\text{fetch}^\mu\text{·sound}]}[\&2]$$

%mode $+\mathcal{D}_1 \;\wedge\; +\mathcal{D}_2 \;\Rightarrow\; -\mathcal{D}_3 \;\wedge\; -\mathcal{D}_4$ [fetch$^\mu$·sound]
%worlds $()\;\mathcal{D}_1 \wedge \mathcal{D}_2 \Rightarrow \mathcal{D}_3 \wedge \mathcal{D}_4$ [fetch$^\mu$·sound]
%total $(\mathcal{D}_2)\;\mathcal{D}_1 \wedge \mathcal{D}_2 \Rightarrow \mathcal{D}_3 \wedge \mathcal{D}_4$ [fetch$^\mu$·sound] □

%theorem $\tilde{\sigma}_1 \rightsquigarrow \tilde{\sigma}_2 \;\wedge\; \tilde{\sigma}_1{}^\diamond=\sigma_1 \;\Rightarrow\; \sigma_1 \rightsquigarrow \sigma_2 \;\wedge\; \tilde{\sigma}_2{}^\diamond=\sigma_2$ [soundness]

Proof.

$$\dfrac{\overset{\mathcal{D}_{111}}{\mathcal{I}(l)=k} \;\wedge\; \dfrac{\overset{\mathcal{D}_{112}}{n\dot{-}k=g}\quad \overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}(g)=(t,n',\mathcal{I}',\mathcal{I}'_\mu,\tilde{\mathcal{E}}',\tilde{\mathcal{E}}'_\mu)}}{\tilde{\mathcal{E}}(n\dot{-}k)=(t,n',\mathcal{I}',\mathcal{I}'_\mu,\tilde{\mathcal{E}}',\tilde{\mathcal{E}}'_\mu)}[\text{i·compute}_2] \;\wedge\; \overset{\mathcal{D}_{211}}{\textit{flatten}\;n\;\tilde{\mathcal{E}}\;\mathcal{I}=\mathcal{L}} \;\Rightarrow\; \overset{\mathcal{D}_{31}}{(t,n',\mathcal{I}',\mathcal{I}'_\mu,\tilde{\mathcal{E}}',\tilde{\mathcal{E}}'_\mu)^\diamond=(t',\mathcal{L}',\mathcal{L}'_\mu,\mathcal{E}'_\mu)} \;\wedge\; \overset{\mathcal{D}_4}{\mathcal{L}(l)=(t',\mathcal{L}',\mathcal{L}'_\mu,\mathcal{E}'_\mu)}\quad[\text{fetch·sound}]}{\dfrac{\dfrac{\overset{\mathcal{D}_{111}}{\mathcal{I}(l)=k}\quad\overset{\mathcal{D}_{112}}{n\dot{-}k=g}}{n\dot{-}\mathcal{I}(l)=g}[\text{compute}_1]\quad \overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}(g)=(t,n',\mathcal{I}',\mathcal{I}'_\mu,\tilde{\mathcal{E}}',\tilde{\mathcal{E}}'_\mu)}}{\langle l,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t,n',\mathcal{I}',\mathcal{I}'_\mu,\tilde{\mathcal{E}}',\tilde{\mathcal{E}}'_\mu,\tilde{\mathcal{S}}\rangle}[\text{i·var}] \;\wedge\; \dfrac{\dfrac{\overset{\mathcal{D}_{211}}{\textit{flatten}\;n\;\tilde{\mathcal{E}}\;\mathcal{I}=\mathcal{L}}\quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\;n\;\tilde{\mathcal{E}})\;\mathcal{I}_\mu=\mathcal{L}_\mu}\quad \overset{\vdots}{\tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu}}{(l,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu)^\diamond=(l,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu)}[\text{clos}^\diamond]\quad \overset{\mathcal{D}_{22}}{\tilde{\mathcal{S}}^\diamond=\mathcal{S}}}{\langle l,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle^\diamond=\langle l,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu,\mathcal{S}\rangle}[\text{state}^\diamond] \;\Rightarrow\; \dfrac{\overset{\mathcal{D}_4}{\mathcal{L}(l)=(t',\mathcal{L}',\mathcal{L}'_\mu,\mathcal{E}'_\mu)}}{\langle l,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu,\mathcal{S}\rangle \rightsquigarrow \langle t',\mathcal{L}',\mathcal{L}'_\mu,\mathcal{E}'_\mu,\mathcal{S}\rangle}[\text{k·var}] \;\wedge\; \dfrac{\overset{\mathcal{D}_{31}}{(t,n',\mathcal{I}',\mathcal{I}'_\mu,\tilde{\mathcal{E}}',\tilde{\mathcal{E}}'_\mu)^\diamond=(t',\mathcal{L}',\mathcal{L}'_\mu,\mathcal{E}'_\mu)}\quad \overset{\mathcal{D}_{22}}{\tilde{\mathcal{S}}^\diamond=\mathcal{S}}}{\langle t,n',\mathcal{I}',\mathcal{I}'_\mu,\tilde{\mathcal{E}}',\tilde{\mathcal{E}}'_\mu,\tilde{\mathcal{S}}\rangle^\diamond=\langle t',\mathcal{L}',\mathcal{L}'_\mu,\mathcal{E}'_\mu,\mathcal{S}\rangle}[\text{state}^\diamond]\quad[\text{soundness}]}[\&1]$$

$$\dfrac{}{\langle (t\,u),n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,(u,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu)::\tilde{\mathcal{S}}\rangle\,[\text{i·app}] \;\wedge\; \dfrac{\dfrac{\overset{\mathcal{D}_{211}}{\textit{flatten}\;n\;\tilde{\mathcal{E}}\;\mathcal{I}=\mathcal{L}}\quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\;n\;\tilde{\mathcal{E}})\;\mathcal{I}_\mu=\mathcal{L}_\mu}\quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu}}{((t\,u),n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu)^\diamond=((t\,u),\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu)}[\text{clos}^\diamond]\quad \overset{\mathcal{D}_{23}}{\tilde{\mathcal{S}}^\diamond=\mathcal{S}}}{\langle (t\,u),n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle^\diamond=\langle (t\,u),\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu,\mathcal{S}\rangle}[\text{state}^\diamond] \;\Rightarrow\; \langle (t\,u),\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu,\mathcal{S}\rangle \rightsquigarrow \langle t,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu,(u,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu)::\mathcal{S}\rangle\,[\text{k·app}] \;\wedge\; \dfrac{\dfrac{\overset{\mathcal{D}_{211}}{\textit{flatten}\;n\;\tilde{\mathcal{E}}\;\mathcal{I}=\mathcal{L}}\quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\;n\;\tilde{\mathcal{E}})\;\mathcal{I}_\mu=\mathcal{L}_\mu}\quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu}}{(t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu)^\diamond=(t,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu)}[\text{clos}^\diamond]\quad \dfrac{\dfrac{\overset{\mathcal{D}_{211}}{\textit{flatten}\;n\;\tilde{\mathcal{E}}\;\mathcal{I}=\mathcal{L}}\quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\;n\;\tilde{\mathcal{E}})\;\mathcal{I}_\mu=\mathcal{L}_\mu}\quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu}}{(u,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu)^\diamond=(u,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu)}[\text{clos}^\diamond]\quad \overset{\mathcal{D}_{23}}{\tilde{\mathcal{S}}^\diamond=\mathcal{S}}}{((u,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu)::\tilde{\mathcal{S}})^\diamond=(u,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu)::\mathcal{S}}[\text{stack}^\diamond_2]}{\langle t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,(u,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu)::\tilde{\mathcal{S}}\rangle^\diamond=\langle t,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu,(u,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu)::\mathcal{S}\rangle}[\text{state}^\diamond]\quad[\text{soundness}]}[\&2]$$

$$\dfrac{\begin{array}{c} n+1\colon \textit{index}\;\cdot\;\overset{\mathcal{D}_1}{(n+1)\dot{-}(n+1)=0}\quad[\text{minus·equals}]\\ \tilde{c}\colon \textit{clos}\;\cdot\;\overset{\mathcal{D}_{211}}{\textit{flatten}\;n\;\tilde{\mathcal{E}}\;\mathcal{I}=\mathcal{L}} \;\Rightarrow\; \overset{\mathcal{D}_2}{\textit{flatten}\;(n+1)\;(\tilde{c};\tilde{\mathcal{E}})\;\mathcal{I}=\mathcal{L}}\quad[\text{weaken·flatten}]\\ \tilde{c}\colon \textit{clos}\;\cdot\;\overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\;n\;\tilde{\mathcal{E}})\;\mathcal{I}_\mu=\mathcal{L}_\mu} \;\Rightarrow\; \overset{\mathcal{D}_3}{\textit{map}\;(\textit{flatten}\;(n+1)\;(\tilde{c};\tilde{\mathcal{E}}))\;\mathcal{I}_\mu=\mathcal{L}_\mu}\quad[\text{weaken·map}]\end{array}}{\langle \lambda t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{c}::\tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t,n+1,(n+1::\mathcal{I}),\mathcal{I}_\mu,(\tilde{c};\tilde{\mathcal{E}}),\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle\,[\text{i·abs}] \;\wedge\; \dfrac{\dfrac{\overset{\mathcal{D}_{211}}{\textit{flatten}\;n\;\tilde{\mathcal{E}}\;\mathcal{I}=\mathcal{L}}\quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\;n\;\tilde{\mathcal{E}})\;\mathcal{I}_\mu=\mathcal{L}_\mu}\quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu}}{(\lambda t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu)^\diamond=(\lambda t,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu)}[\text{clos}^\diamond]\quad \dfrac{\overset{\mathcal{D}_{221}}{\tilde{c}^\diamond=c}\quad \overset{\mathcal{D}_{222}}{\tilde{\mathcal{S}}^\diamond=\mathcal{S}}}{(\tilde{c}::\tilde{\mathcal{S}})^\diamond=c::\mathcal{S}}[\text{stack}^\diamond_2]}{\langle \lambda t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{c}::\tilde{\mathcal{S}}\rangle^\diamond=\langle \lambda t,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu,c::\mathcal{S}\rangle}[\text{state}^\diamond] \;\Rightarrow\; \langle \lambda t,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu,c::\mathcal{S}\rangle \rightsquigarrow \langle t,(c;\mathcal{L}),\mathcal{L}_\mu,\mathcal{E}_\mu,\mathcal{S}\rangle\,[\text{k·abs}] \;\wedge\; \dfrac{\dfrac{\dfrac{\dfrac{\overset{\mathcal{D}_1}{(n+1)\dot{-}(n+1)=0}\quad (\tilde{c};\tilde{\mathcal{E}})(0)=\tilde{c}\,[\text{i·fetch}_1]}{(\tilde{c};\tilde{\mathcal{E}})((n+1)\dot{-}(n+1))=\tilde{c}}[\text{i·compute}_1]\quad \overset{\mathcal{D}_{221}}{\tilde{c}^\diamond=c}\quad \overset{\mathcal{D}_2}{\textit{flatten}\;(n+1)\;(\tilde{c};\tilde{\mathcal{E}})\;\mathcal{I}=\mathcal{L}}}{\textit{flatten}\;(n+1)\;(\tilde{c};\tilde{\mathcal{E}})\;(n+1::\mathcal{I})=(c;\mathcal{L})}[\text{flatten}_2]\quad \overset{\mathcal{D}_3}{\textit{map}\;(\textit{flatten}\;(n+1)\;(\tilde{c};\tilde{\mathcal{E}}))\;\mathcal{I}_\mu=\mathcal{L}_\mu}\quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu}}{(t,n+1,(n+1::\mathcal{I}),\mathcal{I}_\mu,(\tilde{c};\tilde{\mathcal{E}}),\tilde{\mathcal{E}}_\mu)^\diamond=(t,(c;\mathcal{L}),\mathcal{L}_\mu,\mathcal{E}_\mu)}[\text{clos}^\diamond]\quad \overset{\mathcal{D}_{222}}{\tilde{\mathcal{S}}^\diamond=\mathcal{S}}}{\langle t,n+1,(n+1::\mathcal{I}),\mathcal{I}_\mu,(\tilde{c};\tilde{\mathcal{E}}),\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle^\diamond=\langle t,(c;\mathcal{L}),\mathcal{L}_\mu,\mathcal{E}_\mu,\mathcal{S}\rangle}[\text{state}^\diamond]\quad[\text{soundness}]}[\&3]$$

$$\dfrac{}{\langle \textbf{get-context}\;t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t,n,\mathcal{I},(\mathcal{I}::\mathcal{I}_\mu),\tilde{\mathcal{E}},(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu),\tilde{\mathcal{S}}\rangle\,[\text{i·catch}] \;\wedge\; \dfrac{\dfrac{\overset{\mathcal{D}_{211}}{\textit{flatten}\;n\;\tilde{\mathcal{E}}\;\mathcal{I}=\mathcal{L}}\quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\;n\;\tilde{\mathcal{E}})\;\mathcal{I}_\mu=\mathcal{L}_\mu}\quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu}}{(\textbf{get-context}\;t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu)^\diamond=(\textbf{get-context}\;t,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu)}[\text{clos}^\diamond]\quad \overset{\mathcal{D}_{22}}{\tilde{\mathcal{S}}^\diamond=\mathcal{S}}}{\langle \textbf{get-context}\;t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle^\diamond=\langle \textbf{get-context}\;t,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu,\mathcal{S}\rangle}[\text{state}^\diamond] \;\Rightarrow\; \langle \textbf{get-context}\;t,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu,\mathcal{S}\rangle \rightsquigarrow \langle t,\mathcal{L},(\mathcal{L};\mathcal{L}_\mu),(\mathcal{S};\mathcal{E}_\mu),\mathcal{S}\rangle\,[\text{k·catch}] \;\wedge\; \dfrac{\dfrac{\overset{\mathcal{D}_{211}}{\textit{flatten}\;n\;\tilde{\mathcal{E}}\;\mathcal{I}=\mathcal{L}}\quad \dfrac{\overset{\mathcal{D}_{211}}{\textit{flatten}\;n\;\tilde{\mathcal{E}}\;\mathcal{I}=\mathcal{L}}\quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\;n\;\tilde{\mathcal{E}})\;\mathcal{I}_\mu=\mathcal{L}_\mu}}{\textit{map}\;(\textit{flatten}\;n\;\tilde{\mathcal{E}})\;(\mathcal{I}::\mathcal{I}_\mu)=(\mathcal{L};\mathcal{L}_\mu)}[\text{map}_2]\quad \dfrac{\overset{\mathcal{D}_{22}}{\tilde{\mathcal{S}}^\diamond=\mathcal{S}}\quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu}}{(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu)^\diamond=(\mathcal{S};\mathcal{E}_\mu)}[\text{k·env}^\diamond_2]}{(t,n,\mathcal{I},(\mathcal{I}::\mathcal{I}_\mu),\tilde{\mathcal{E}},(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu))^\diamond=(t,\mathcal{L},(\mathcal{L};\mathcal{L}_\mu),(\mathcal{S};\mathcal{E}_\mu))}[\text{clos}^\diamond]\quad \overset{\mathcal{D}_{22}}{\tilde{\mathcal{S}}^\diamond=\mathcal{S}}}{\langle t,n,\mathcal{I},(\mathcal{I}::\mathcal{I}_\mu),\tilde{\mathcal{E}},(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu),\tilde{\mathcal{S}}\rangle^\diamond=\langle t,\mathcal{L},(\mathcal{L};\mathcal{L}_\mu),(\mathcal{S};\mathcal{E}_\mu),\mathcal{S}\rangle}[\text{state}^\diamond]\quad[\text{soundness}]}[\&4]$$

$$\dfrac{\begin{array}{c}\overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu} \wedge \overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}_\mu(\alpha)=\tilde{\mathcal{S}}'} \Rightarrow \overset{\mathcal{D}_{32}}{\tilde{\mathcal{S}}'^\diamond=\mathcal{S}'} \wedge \overset{\mathcal{D}_4}{\mathcal{E}_\mu(\alpha)=\mathcal{S}'}\quad[\text{fetch}^\mu\text{·sound}]\\ \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\;n\;\tilde{\mathcal{E}})\;\mathcal{I}_\mu=\mathcal{L}_\mu} \wedge \overset{\mathcal{D}_{11}}{\mathcal{I}_\mu(\alpha)=\mathcal{I}'} \Rightarrow \overset{\mathcal{D}_5}{\textit{flatten}\;n\;\tilde{\mathcal{E}}\;\mathcal{I}'=\mathcal{L}'} \wedge \overset{\mathcal{D}_6}{\mathcal{L}_\mu(\alpha)=\mathcal{L}'}\quad[\text{map·sound}]\end{array}}{\dfrac{\overset{\mathcal{D}_{11}}{\mathcal{I}_\mu(\alpha)=\mathcal{I}'}\quad \overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}_\mu(\alpha)=\tilde{\mathcal{S}}'}}{\langle \textbf{set-context}\;\alpha\;t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t,n,\mathcal{I}',\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}'\rangle}[\text{i·throw}] \;\wedge\; \dfrac{\dfrac{\overset{\vdots}{\textit{flatten}\;n\;\tilde{\mathcal{E}}\;\mathcal{I}=\mathcal{L}}\quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\;n\;\tilde{\mathcal{E}})\;\mathcal{I}_\mu=\mathcal{L}_\mu}\quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu}}{(\textbf{set-context}\;\alpha\;t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu)^\diamond=(\textbf{set-context}\;\alpha\;t,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu)}[\text{clos}^\diamond]\quad \overset{\vdots}{\tilde{\mathcal{S}}^\diamond=\mathcal{S}}}{\langle \textbf{set-context}\;\alpha\;t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle^\diamond=\langle \textbf{set-context}\;\alpha\;t,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu,\mathcal{S}\rangle}[\text{state}^\diamond] \;\Rightarrow\; \dfrac{\overset{\mathcal{D}_6}{\mathcal{L}_\mu(\alpha)=\mathcal{L}'}\quad \overset{\mathcal{D}_4}{\mathcal{E}_\mu(\alpha)=\mathcal{S}'}}{\langle \textbf{set-context}\;\alpha\;t,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu,\mathcal{S}\rangle \rightsquigarrow \langle t,\mathcal{L}',\mathcal{L}_\mu,\mathcal{E}_\mu,\mathcal{S}'\rangle}[\text{k·throw}] \;\wedge\; \dfrac{\dfrac{\overset{\mathcal{D}_5}{\textit{flatten}\;n\;\tilde{\mathcal{E}}\;\mathcal{I}'=\mathcal{L}'}\quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\;n\;\tilde{\mathcal{E}})\;\mathcal{I}_\mu=\mathcal{L}_\mu}\quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu}}{(t,n,\mathcal{I}',\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu)^\diamond=(t,\mathcal{L}',\mathcal{L}_\mu,\mathcal{E}_\mu)}[\text{clos}^\diamond]\quad \overset{\mathcal{D}_{32}}{\tilde{\mathcal{S}}'^\diamond=\mathcal{S}'}}{\langle t,n,\mathcal{I}',\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}'\rangle^\diamond=\langle t,\mathcal{L}',\mathcal{L}_\mu,\mathcal{E}_\mu,\mathcal{S}'\rangle}[\text{state}^\diamond]\quad[\text{soundness}]}[\&5]$$

%mode $+\mathcal{D}_1 \;\wedge\; +\mathcal{D}_2 \;\Rightarrow\; -\mathcal{D}_3 \;\wedge\; -\mathcal{D}_4$ [soundness]
%worlds $()\;\mathcal{D}_1 \wedge \mathcal{D}_2 \Rightarrow \mathcal{D}_3 \wedge \mathcal{D}_4$ [soundness]
%terminates $(\mathcal{D}_1)\;\mathcal{D}_1 \wedge \mathcal{D}_2 \Rightarrow \mathcal{D}_3 \wedge \mathcal{D}_4$ [soundness]
%total $(\mathcal{D}_1)\;\mathcal{D}_1 \wedge \mathcal{D}_2 \Rightarrow \mathcal{D}_3 \wedge \mathcal{D}_4$ [soundness] □