1 Krivine abstract machine with catch/throw

1.1 Syntax

1.1.1 Index, indices and tables

%datatype *index*
%name *index* n α

$$\begin{array}{lcl} n & ::= & 0 \\ & | & n+1 \end{array}$$

%datatype *vector*
%name *vector* $\mathcal{I}$

$$\begin{array}{lcl} \mathcal{I} & ::= & [\,] \\ & | & n :: \mathcal{I} \end{array}$$

%datatype *table*
%name *table* $\mathcal{I}_\mu$

$$\begin{array}{lcl} \mathcal{I}_\mu & ::= & [\,] \\ & | & \mathcal{I} :: \mathcal{I}_\mu \end{array}$$

1.1.2 Term

%datatype *term*
%name *term* t

$$\begin{array}{lcl} t & ::= & n \\ & | & t_1\, t_2 \\ & | & \lambda t \\ & | & \mathbf{catch}\, t \\ & | & \mathbf{throw}\, \alpha\, t \end{array}$$

Remark. Syntax of safe λ_{ct}-terms:

$$\begin{array}{lcl} t & ::= & n \\ & | & t_1\, t_2 \\ & | & \lambda t \\ & | & \mathbf{get\text{-}context}\, t \\ & | & \mathbf{set\text{-}context}\, \alpha\, t \end{array}$$

1.2 Subtraction

%judgment $n_1 \dot{-} n_2 = n_3$

$$n_1 \dot{-} 0 = n_1 \;^{[\mathrm{minus}_1]}$$

$$(n_1+1) \dot{-} (n_2+1) = n_3 \;^{[\mathrm{minus}_2]} \quad \textit{when} \quad n_1 \dot{-} n_2 = n_3$$

%mode $+n_1 \dot{-} +n_2 = -n_3$
%worlds $()$ $n_1 \dot{-} n_2 = n_3$

%terminates (n_1) $n_1 \dot{-} n_2 = n_3$
%unique $+n_1 \dot{-} +n_2 = -1n_3$

%lemma $n_1 \dot{-} n_2 = n_3 \quad \Rightarrow \quad n_1 \dot{-} n_3 = n_2$ [minus-swap]

1.2.1 Fetch (indices)

%judgment $\mathcal{I}(n_1) = n_2$

$$(n :: \mathcal{I})(0) = n \text{ [fetch}_1^{\mathcal{I}}]$$
$$(n :: \mathcal{I})(n_1 + 1) = n_2 \text{ [fetch}_2^{\mathcal{I}}] \quad \textit{when} \quad \mathcal{I}(n_1) = n_2$$

%mode $+\mathcal{I}(+n_1) = -n_2$
%worlds $()$ $\mathcal{I}(n_1) = n_2$
%terminates n_1 $\mathcal{I}(n_1) = n_2$
%unique $+\mathcal{I}(+n_1) = -1n_2$

1.2.2 Fetch (table)

%judgment $\mathcal{I}_\mu(n) = \mathcal{I}$

$$(\mathcal{I} :: \mathcal{I}_\mu)(0) = \mathcal{I}^{\left[\text{fetch}_1^{\mathcal{I}_\mu}\right]}$$
$$(\mathcal{I}' :: \mathcal{I}_\mu)(\alpha + 1) = \mathcal{I}^{\left[\text{fetch}_2^{\mathcal{I}_\mu}\right]} \quad \textit{when} \quad \mathcal{I}_\mu(\alpha) = \mathcal{I}$$

%mode $+\mathcal{I}_\mu(+\alpha) = -\mathcal{I}$
%worlds $()$ $\mathcal{I}_\mu(\alpha) = \mathcal{I}$
%terminates α $\mathcal{I}_\mu(\alpha) = \mathcal{I}$
%unique $+\mathcal{I}_\mu(+\alpha) = -1\mathcal{I}$

1.2.3 Compute

%judgment $n_1 \dot{-} \mathcal{I}(n_2) = n_3$

$$n \dot{-} \mathcal{I}(l) = g \text{ [compute}_1] \quad \textit{when} \quad \mathcal{I}(l) = k, \quad n \dot{-} k = g$$

%mode $+n_1 \dot{-} +\mathcal{I}(+n_2) = -n_3$
%worlds $()$ $n_1 \dot{-} \mathcal{I}(n_2) = n_3$
%terminates $\{\}$ $n_1 \dot{-} \mathcal{I}(n_2) = n_3$
%unique $+n_1 \dot{-} +\mathcal{I}(+n_2) = -1n_3$

2 Safe $\boldsymbol{\lambda}_{\mathbf{ct}}$-terms

2.1 Safety

%judgment $n \in \mathcal{I}$

$$n \in (n :: \mathcal{I}) \quad \text{[member}_1]$$
$$n \in (n' :: \mathcal{I}) \quad \text{[member}_2] \quad \textit{when} \quad n \in \mathcal{I}$$

%mode $+n \in +\mathcal{I}$

%worlds $()$ $\quad n \in \mathcal{I}$
%terminates $\mathcal{I}$ $\quad n \in \mathcal{I}$

%lemma $k \in \mathcal{I} \quad \Rightarrow \quad \mathcal{I}(n) = k \quad \textit{for some } n$ [domain]

%judgment $\mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(t)$

$$
\begin{array}{llll}
\mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(g) & [\text{safe}_1] & \textit{when} & n \dot{-} g = k, \quad k \in \mathcal{I} \\
\mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(t\,u) & [\text{safe}_2] & \textit{when} & \mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(t), \quad \mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(u) \\
\mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(\lambda t) & [\text{safe}_3] & \textit{when} & \mathit{Safe}_{n+1}^{(n+1\,::\,\mathcal{I}),\mathcal{I}_\mu}(t) \\
\mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(\mathbf{catch}\, t) & [\text{safe}_4] & \textit{when} & \mathit{Safe}_n^{\mathcal{I},(\mathcal{I}\,::\,\mathcal{I}_\mu)}(t) \\
\mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(\mathbf{throw}\, \alpha\, t) & [\text{safe}_5] & \textit{when} & \mathcal{I}_\mu(\alpha) = \mathcal{I}', \quad \mathit{Safe}_n^{\mathcal{I}',\mathcal{I}_\mu}(t)
\end{array}
$$

%mode $\mathit{Safe}_{+n}^{+\mathcal{I},+\mathcal{I}_\mu}(+t)$
%worlds $()$ $\quad \mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(t)$
%terminates t $\quad \mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(t)$

2.2 From local indices to global indices

%judgment $\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t_1) = t_2$

$$
\begin{array}{llll}
\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(l) = g & [\downarrow_1] & \textit{when} & n \dot{-} \mathcal{I}(l) = g \\
\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t\,u) = t'\,u' & [\downarrow_2] & \textit{when} & \downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t) = t', \quad \downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(u) = u' \\
\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(\lambda t) = \lambda t' & [\downarrow_3] & \textit{when} & \downarrow_{n+1}^{(n+1\,::\,\mathcal{I}),\mathcal{I}_\mu}(t) = t' \\
\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(\textbf{get-context}\, t) = \mathbf{catch}\, t' & [\downarrow_4] & \textit{when} & \downarrow_n^{\mathcal{I},(\mathcal{I}\,::\,\mathcal{I}_\mu)}(t) = t' \\
\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(\textbf{set-context}\, \alpha\, t) = \mathbf{throw}\, \alpha\, t' & [\downarrow_5] & \textit{when} & \mathcal{I}_\mu(\alpha) = \mathcal{I}', \quad \downarrow_n^{\mathcal{I}',\mathcal{I}_\mu}(t) = t'
\end{array}
$$

%mode $\downarrow_{+n}^{+\mathcal{I},+\mathcal{I}_\mu}(+t) = -t'$
%worlds $()$ $\quad \downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t) = t'$
%terminates t $\quad \downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t) = t'$
%unique $\downarrow_{+n}^{+\mathcal{I},+\mathcal{I}_\mu}(+t) = -1t'$

%lemma $\mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(t') \quad \Rightarrow \quad \downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t) = t' \quad \textit{for some } t$ [safe·image]