1 First simulation (completeness)

1.1 Syntax

1.1.1 Index, indices and tables

%datatype *index*
%name *index* n α

$$\begin{array}{lcl} n & ::= & 0 \\ & \mid & n+1 \end{array}$$

%datatype *vector*
%name *vector* $\mathcal{I}$

$$\begin{array}{lcl} \mathcal{I} & ::= & [\,] \\ & \mid & n :: \mathcal{I} \end{array}$$

%datatype *table*
%name *table* $\mathcal{I}_\mu$

$$\begin{array}{lcl} \mathcal{I}_\mu & ::= & [\,] \\ & \mid & \mathcal{I} :: \mathcal{I}_\mu \end{array}$$

1.1.2 Term

%datatype *term*
%name *term* t

$$\begin{array}{lcl} t & ::= & n \\ & \mid & t_1\, t_2 \\ & \mid & \lambda t \\ & \mid & \mathbf{catch}\, t \\ & \mid & \mathbf{throw}\, \alpha\, t \end{array}$$

Remark. Syntax of safe λ_{ct}-terms:

$$\begin{array}{lcl} t & ::= & n \\ & \mid & t_1\, t_2 \\ & \mid & \lambda t \\ & \mid & \mathbf{get\text{-}context}\, t \\ & \mid & \mathbf{set\text{-}context}\, \alpha\, t \end{array}$$

1.2 Subtraction

%judgment $n_1 \dot{-} n_2 = n_3$

$$n_1 \dot{-} 0 = n_1 \;{}^{[\text{minus}_1]}$$
$$(n_1+1) \dot{-} (n_2+1) = n_3 \;{}^{[\text{minus}_2]} \quad \textit{when} \quad n_1 \dot{-} n_2 = n_3$$

%mode $+n_1 \dot{-} +n_2 = -n_3$
%worlds $()$ $n_1 \dot{-} n_2 = n_3$
%terminates (n_1) $n_1 \dot{-} n_2 = n_3$
%unique $+n_1 \dot{-} +n_2 = -1 n_3$

1.2.1 Fetch (indices)

%judgment $\mathcal{I}(n_1) = n_2$

$$(n :: \mathcal{I})(0) = n \;{}^{[\text{fetch}_1^{\mathcal{I}}]}$$
$$(n :: \mathcal{I})(n_1+1) = n_2 \;{}^{[\text{fetch}_2^{\mathcal{I}}]} \quad \textit{when} \quad \mathcal{I}(n_1) = n_2$$

%mode $+\mathcal{I}(+n_1) = -n_2$
%worlds $()$ $\mathcal{I}(n_1) = n_2$
%terminates n_1 $\mathcal{I}(n_1) = n_2$
%unique $+\mathcal{I}(+n_1) = -1 n_2$

1.2.2 Fetch (table)

%judgment $\mathcal{I}_\mu(n) = \mathcal{I}$

$$(\mathcal{I} :: \mathcal{I}_\mu)(0) = \mathcal{I} \;{}^{\left[\text{fetch}_1^{\mathcal{I}_\mu}\right]}$$
$$(\mathcal{I}' :: \mathcal{I}_\mu)(\alpha+1) = \mathcal{I} \;{}^{\left[\text{fetch}_2^{\mathcal{I}_\mu}\right]} \quad \textit{when} \quad \mathcal{I}_\mu(\alpha) = \mathcal{I}$$

%mode $+\mathcal{I}_\mu(+\alpha) = -\mathcal{I}$
%worlds $()$ $\mathcal{I}_\mu(\alpha) = \mathcal{I}$
%terminates α $\mathcal{I}_\mu(\alpha) = \mathcal{I}$
%unique $+\mathcal{I}_\mu(+\alpha) = -1\mathcal{I}$

1.2.3 Compute

%judgment $n_1 \dot{-} \mathcal{I}(n_2) = n_3$

$$n \dot{-} \mathcal{I}(l) = g \;{}^{[\text{compute}_1]} \quad \textit{when} \quad \mathcal{I}(l) = k, \quad n \dot{-} k = g$$

%mode $+n_1 \dot{-} +\mathcal{I}(+n_2) = -n_3$
%worlds $()$ $n_1 \dot{-} \mathcal{I}(n_2) = n_3$
%terminates $\{\}$ $n_1 \dot{-} \mathcal{I}(n_2) = n_3$
%unique $+n_1 \dot{-} +\mathcal{I}(+n_2) = -1 n_3$

1.3 From local indices to global indices

%judgment $\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t_1) = t_2$

$$\begin{array}{lll}
\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(l) = g & {}^{[\downarrow_1]} & \textit{when} \quad n \dot{-} \mathcal{I}(l) = g \\
\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t\,u) = t'\,u' & {}^{[\downarrow_2]} & \textit{when} \quad \downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t) = t', \quad \downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(u) = u' \\
\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(\lambda t) = \lambda t' & {}^{[\downarrow_3]} & \textit{when} \quad \downarrow_{n+1}^{(n+1 :: \mathcal{I}),\mathcal{I}_\mu}(t) = t' \\
\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(\mathbf{get\text{-}context}\, t) = \mathbf{catch}\, t' & {}^{[\downarrow_4]} & \textit{when} \quad \downarrow_n^{\mathcal{I},(\mathcal{I} :: \mathcal{I}_\mu)}(t) = t' \\
\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(\mathbf{set\text{-}context}\, \alpha\, t) = \mathbf{throw}\, \alpha\, t' & {}^{[\downarrow_5]} & \textit{when} \quad \mathcal{I}_\mu(\alpha) = \mathcal{I}', \quad \downarrow_n^{\mathcal{I}',\mathcal{I}_\mu}(t) = t'
\end{array}$$

%mode $\downarrow_{+n}^{+\mathcal{I},+\mathcal{I}_\mu}(+t) = -t'$
%worlds $()$ $\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t) = t'$
%terminates t $\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t) = t'$
%unique $\downarrow_{+n}^{+\mathcal{I},+\mathcal{I}_\mu}(+t) = -1t'$

1.3.1 Closure, environment and stack

%datatype *clos* **%name** *clos* c
%datatype *c-env* **%name** *c-env* $\mathcal{E}$
%datatype *k-env* **%name** *k-env* $\mathcal{E}_\mu$
%datatype *stack* **%name** *stack* $\mathcal{S}$

$$\begin{array}{lcl} c & ::= & (t, \mathcal{E}, \mathcal{E}_\mu) \end{array}$$

$$\begin{array}{lcl} \mathcal{E} & ::= & () \\ & \mid & (c; \mathcal{E}) \end{array}$$

$$\begin{array}{lcl} \mathcal{E}_\mu & ::= & () \\ & \mid & (\mathcal{S}; \mathcal{E}_\mu) \end{array}$$

$$\begin{array}{lcl} \mathcal{S} & ::= & [\,] \\ & \mid & c :: \mathcal{S} \end{array}$$

%datatype *state*
%name *state* σ

$$\begin{array}{lcl} \sigma & ::= & \langle t, \mathcal{E}, \mathcal{E}_\mu, \mathcal{S} \rangle \end{array}$$

1.4 Judgments

1.4.1 Fetch a closure

%judgment $\mathcal{E}(n) = c$

$$(c; \mathcal{E})(0) = c \;{}^{[\text{fetch}_1]}$$
$$(c'; \mathcal{E})(n+1) = c \;{}^{[\text{fetch}_2]} \quad \textit{when} \quad \mathcal{E}(n) = c$$

%mode $+\mathcal{E}(+n) = -c$
%worlds $()$ $\mathcal{E}(n) = c$
%terminates $\mathcal{E}$ $\mathcal{E}(n) = c$
%unique $+\mathcal{E}(+n) = -1c$

1.4.2 Fetch a stack

%judgment $\mathcal{E}_\mu(n) = \mathcal{S}$

$$(\mathcal{S}; \mathcal{E}_\mu)(0) = \mathcal{S} \;{}^{[\text{fetch}_1^{\mu}]}$$
$$(\mathcal{S}'; \mathcal{E}_\mu)(n+1) = \mathcal{S} \;{}^{[\text{fetch}_2^{\mu}]} \quad \textit{when} \quad \mathcal{E}_\mu(n) = \mathcal{S}$$

%mode $+\mathcal{E}_\mu(+n) = -\mathcal{S}$
%worlds $()$ $\mathcal{E}_\mu(n) = \mathcal{S}$
%terminates $\mathcal{E}_\mu$ $\mathcal{E}_\mu(n) = \mathcal{S}$
%unique $+\mathcal{E}_\mu(+n) = -1\mathcal{S}$

1.4.3 Evaluation rules

%judgment $\sigma_1 \leadsto \sigma_2$

$$\langle k, \mathcal{E}, \mathcal{E}_\mu, \mathcal{S} \rangle \leadsto \langle t, \mathcal{E}', \mathcal{E}_\mu', \mathcal{S} \rangle \;{}^{[\text{k}\cdot\text{var}]} \quad \textit{when} \quad \mathcal{E}(k) = (t, \mathcal{E}', \mathcal{E}_\mu')$$
$$\langle (t\,u), \mathcal{E}, \mathcal{E}_\mu, \mathcal{S} \rangle \leadsto \langle t, \mathcal{E}, \mathcal{E}_\mu, (u, \mathcal{E}, \mathcal{E}_\mu) :: \mathcal{S} \rangle \;{}^{[\text{k}\cdot\text{app}]}$$
$$\langle \lambda t, \mathcal{E}, \mathcal{E}_\mu, c :: \mathcal{S} \rangle \leadsto \langle t, (c; \mathcal{E}), \mathcal{E}_\mu, \mathcal{S} \rangle \;{}^{[\text{k}\cdot\text{abs}]}$$
$$\langle \mathbf{catch}\, t, \mathcal{E}, \mathcal{E}_\mu, \mathcal{S} \rangle \leadsto \langle t, \mathcal{E}, (\mathcal{S}; \mathcal{E}_\mu), \mathcal{S} \rangle \;{}^{[\text{k}\cdot\text{catch}]}$$
$$\langle \mathbf{throw}\, \alpha\, t, \mathcal{E}, \mathcal{E}_\mu, \mathcal{S} \rangle \leadsto \langle t, \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}' \rangle \;{}^{[\text{k}\cdot\text{throw}]} \quad \textit{when} \quad \mathcal{E}_\mu(\alpha) = \mathcal{S}'$$

%mode $+\sigma_1 \leadsto -\sigma_2$
%worlds $()$ $\sigma_1 \leadsto \sigma_2$
%unique $+\sigma_1 \leadsto -1\sigma_2$

1.5 Abstract machine for safe $\boldsymbol{\lambda}_{\textbf{ct}}$-terms

1.5.1 Syntax

%datatype *clos*
%datatype *c-env*
%datatype *k-env*
%datatype *stack*
%name *clos* $\tilde{c}$
%name *c-env* $\tilde{\mathcal{E}}$
%name *k-env* $\tilde{\mathcal{E}}_\mu$
%name *stack* $\tilde{\mathcal{S}}$

$$\begin{array}{lcl} \tilde{c} & ::= & \left(t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu\right) \end{array}$$

$$\begin{array}{lcl} \tilde{\mathcal{E}} & ::= & () \\ & \mid & (\tilde{c}; \tilde{\mathcal{E}}) \end{array}$$

$$\begin{array}{lcl} \tilde{\mathcal{E}}_\mu & ::= & () \\ & \mid & \left(\tilde{\mathcal{S}}; \tilde{\mathcal{E}}_\mu\right) \end{array}$$

$$\begin{array}{lcl} \tilde{\mathcal{S}} & ::= & [\,] \\ & \mid & \tilde{c} :: \tilde{\mathcal{S}} \end{array}$$

%datatype *state*
%name *state* $\tilde{\sigma}$

$$\begin{array}{lcl} \tilde{\sigma} & ::= & \langle t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}} \rangle \end{array}$$

1.5.2 Fetch a closure

%judgment $\tilde{\mathcal{E}}(n) = \tilde{c}$

$$(\tilde{c}; \tilde{\mathcal{E}})(0) = \tilde{c} \;{}^{[\text{i}\cdot\text{fetch}_1]}$$
$$(\tilde{c}'; \tilde{\mathcal{E}})(n+1) = \tilde{c} \;{}^{[\text{i}\cdot\text{fetch}_2]} \quad \textit{when} \quad \tilde{\mathcal{E}}(n) = \tilde{c}$$

%mode $+\tilde{\mathcal{E}}(+n) = -\tilde{c}$
%worlds $()$ $\tilde{\mathcal{E}}(n) = \tilde{c}$
%terminates $\tilde{\mathcal{E}}$ $\tilde{\mathcal{E}}(n) = \tilde{c}$
%unique $+\tilde{\mathcal{E}}(+n) = -1\tilde{c}$

1.5.3 Fetch a stack

%judgment $\tilde{\mathcal{E}}_\mu(n) = \tilde{\mathcal{S}}$

$$(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu)(0) = \tilde{\mathcal{S}}\ ^{[\text{i}\cdot\text{fetch}^\mu_1]}$$
$$(\tilde{\mathcal{S}}';\tilde{\mathcal{E}}_\mu)(n+1) = \tilde{\mathcal{S}}\ ^{[\text{i}\cdot\text{fetch}^\mu_2]} \quad \textit{when} \quad \tilde{\mathcal{E}}_\mu(n) = \tilde{\mathcal{S}}$$

%mode $+\tilde{\mathcal{E}}_\mu(+n) = -\tilde{\mathcal{S}}$
%worlds $()\quad \tilde{\mathcal{E}}_\mu(n) = \tilde{\mathcal{S}}$
%terminates $\tilde{\mathcal{E}}_\mu \quad \tilde{\mathcal{E}}_\mu(n) = \tilde{\mathcal{S}}$
%unique $+\tilde{\mathcal{E}}_\mu(+n) = -1\tilde{\mathcal{S}}$

2 Evaluation rules

%judgment $\tilde{\sigma}_1 \rightsquigarrow \tilde{\sigma}_2$

$$\langle l, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t, n', \mathcal{I}', \mathcal{I}'_\mu, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_\mu, \tilde{\mathcal{S}}\rangle\ ^{[\text{i}\cdot\text{var}]}$$
$$\textit{when} \quad n \dot{-} \mathcal{I}(l) = g, \quad \tilde{\mathcal{E}}(g) = (t, n', \mathcal{I}', \mathcal{I}'_\mu, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_\mu)$$
$$\langle (t\,u), n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, (u, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu) :: \tilde{\mathcal{S}}\rangle\ ^{[\text{i}\cdot\text{app}]}$$
$$\langle \lambda t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{c} :: \tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t, n+1, (n+1 :: \mathcal{I}), \mathcal{I}_\mu, (\tilde{c};\tilde{\mathcal{E}}), \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle\ ^{[\text{i}\cdot\text{abs}]}$$
$$\langle \textbf{get-context}\, t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t, n, \mathcal{I}, (\mathcal{I} :: \mathcal{I}_\mu), \tilde{\mathcal{E}}, (\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu), \tilde{\mathcal{S}}\rangle\ ^{[\text{i}\cdot\text{catch}]}$$
$$\langle \textbf{set-context}\, \alpha\, t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t, n, \mathcal{I}', \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}'\rangle\ ^{[\text{i}\cdot\text{throw}]}$$
$$\textit{when} \quad \mathcal{I}_\mu(\alpha) = \mathcal{I}', \quad \tilde{\mathcal{E}}_\mu(\alpha) = \tilde{\mathcal{S}}'$$

%mode $+\sigma_1 \rightsquigarrow -\sigma_2$
%worlds $()\quad \sigma_1 \rightsquigarrow \sigma_2$
%unique $+\sigma_1 \rightsquigarrow -1\sigma_2$

3 Translation

%judgment $\tilde{c}^\star = c$
%judgment $\tilde{\mathcal{S}}^\star = \mathcal{S}$
%judgment $\tilde{\mathcal{E}}^\star = \mathcal{E}$
%judgment $\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu$

$$(t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu)^\star = (u, \mathcal{E}, \mathcal{E}_\mu)\ ^{[\text{clos}^\star]} \quad \textit{when} \quad \downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t) = u, \quad \tilde{\mathcal{E}}^\star = \mathcal{E}, \quad \tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu$$

$$[]^\star = []\ ^{[\text{stack}^\star_1]}$$
$$(\tilde{c} :: \tilde{\mathcal{S}})^\star = c :: \mathcal{S}\ ^{[\text{stack}^\star_2]} \quad \textit{when} \quad \tilde{c}^\star = c, \quad \tilde{\mathcal{S}}^\star = \mathcal{S}$$

$$()^\star = ()\ ^{[\text{c}\cdot\text{env}^\star_1]}$$
$$(\tilde{c};\tilde{\mathcal{E}})^\star = (c;\mathcal{E})\ ^{[\text{c}\cdot\text{env}^\star_2]} \quad \textit{when} \quad \tilde{c}^\star = c, \quad \tilde{\mathcal{E}}^\star = \mathcal{E}$$

$$()^\star = ()\ ^{[\text{k}\cdot\text{env}^\star_1]}$$
$$(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu)^\star = (\mathcal{S};\mathcal{E}_\mu)\ ^{[\text{k}\cdot\text{env}^\star_2]} \quad \textit{when} \quad \tilde{\mathcal{S}}^\star = \mathcal{S}, \quad \tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu$$

%mode
$+\tilde{c}^\star = -c$
$+\tilde{\mathcal{S}}^\star = -\mathcal{S}$
$+\tilde{\mathcal{E}}^\star = -\mathcal{E}$
$+\tilde{\mathcal{E}}_\mu{}^\star = -\mathcal{E}_\mu$

%worlds $()$
$\tilde{c}^\star = c$
$\tilde{\mathcal{S}}^\star = \mathcal{S}$
$\tilde{\mathcal{E}}^\star = \mathcal{E}$
$\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu$

%terminates $(\tilde{c}\ \tilde{\mathcal{S}}\ \tilde{\mathcal{E}}\ \tilde{\mathcal{E}}_\mu)$
$\tilde{c}^\star = c$
$\tilde{\mathcal{S}}^\star = \mathcal{S}$
$\tilde{\mathcal{E}}^\star = \mathcal{E}$
$\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu$

%unique
$+\tilde{c}^\star = -1c$
$+\tilde{\mathcal{S}}^\star = -1\mathcal{S}$
$+\tilde{\mathcal{E}}^\star = -1\mathcal{E}$
$+\tilde{\mathcal{E}}_\mu{}^\star = -1\mathcal{E}_\mu$

%judgment $\tilde{\sigma}^\star = \sigma$
$$\langle t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle^\star = \langle u, \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}\rangle\ ^{[\text{state}^\star]} \quad \textit{when} \quad (t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu)^\star = (u, \mathcal{E}, \mathcal{E}_\mu), \quad \tilde{\mathcal{S}}^\star = \mathcal{S}$$

%mode $+\tilde{\sigma}^\star = -\sigma$
%worlds $()\quad \tilde{\sigma}^\star = \sigma$
%unique $+\tilde{\sigma}^\star = -1\sigma$

4 Completeness

%lemma $\tilde{\mathcal{E}}^\star = \mathcal{E} \quad \wedge \quad \mathcal{E}(n) = c \quad \Rightarrow \quad \tilde{\mathcal{E}}(n) = \tilde{c} \quad \wedge \quad \tilde{c}^\star = c \quad \textit{for some } \tilde{c}$ [fetch·complete]

Proof.

$$\dfrac{}{\dfrac{\overset{\mathcal{D}_{11}}{\tilde{c}^\star = c} \quad \overset{\vdots}{\tilde{\mathcal{E}}^\star = \mathcal{E}}}{(\tilde{c};\tilde{\mathcal{E}})^\star = (c;\mathcal{E})}\,[\text{c}\cdot\text{env}^\star_2] \quad \wedge \quad (c;\mathcal{E})(0) = c\ ^{[\text{fetch}_1]} \quad \Rightarrow \quad (\tilde{c};\tilde{\mathcal{E}})(0) = \tilde{c}\ ^{[\text{i}\cdot\text{fetch}_1]} \quad \wedge \quad \overset{\mathcal{D}_{11}}{\tilde{c}^\star = c} \quad ^{[\text{fetch}\cdot\text{complete}]}}\,[\&1]$$

$$\dfrac{\overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}^\star = \mathcal{E}} \quad \wedge \quad \overset{\mathcal{D}_4}{\mathcal{E}(n) = c} \quad \Rightarrow \quad \overset{\mathcal{D}_2}{\tilde{\mathcal{E}}(n) = \tilde{c}} \quad \wedge \quad \overset{\mathcal{D}_3}{\tilde{c}^\star = c} \quad ^{[\text{fetch}\cdot\text{complete}]}}{\dfrac{\overset{\vdots}{\tilde{c}'^\star = c'} \quad \overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}^\star = \mathcal{E}}}{(\tilde{c}';\tilde{\mathcal{E}})^\star = (c';\mathcal{E})}\,[\text{c}\cdot\text{env}^\star_2] \quad \wedge \quad \dfrac{\overset{\mathcal{D}_4}{\mathcal{E}(n) = c}}{(c';\mathcal{E})(n+1) = c}\,[\text{fetch}_2] \quad \Rightarrow \quad \dfrac{\overset{\mathcal{D}_2}{\tilde{\mathcal{E}}(n) = \tilde{c}}}{(\tilde{c}';\tilde{\mathcal{E}})(n+1) = \tilde{c}}\,[\text{i}\cdot\text{fetch}_2] \quad \wedge \quad \overset{\mathcal{D}_3}{\tilde{c}^\star = c} \quad ^{[\text{fetch}\cdot\text{complete}]}}\,[\&2]$$

%mode $+\mathcal{D}_1 \quad \wedge \quad +\mathcal{D}_2 \quad \Rightarrow \quad -\mathcal{D}_3 \quad \wedge \quad -\mathcal{D}_4$ [fetch·complete]
%worlds $()\quad \mathcal{D}_1 \quad \wedge \quad \mathcal{D}_2 \quad \Rightarrow \quad \mathcal{D}_3 \quad \wedge \quad \mathcal{D}_4$ [fetch·complete]
%total $(\mathcal{D}_2)\quad \mathcal{D}_1 \quad \wedge \quad \mathcal{D}_2 \quad \Rightarrow \quad \mathcal{D}_3 \quad \wedge \quad \mathcal{D}_4$ [fetch·complete]

□

%lemma $\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu \quad \wedge \quad \mathcal{E}_\mu(\alpha) = \mathcal{S} \quad \Rightarrow \quad \tilde{\mathcal{E}}_\mu(\alpha) = \tilde{\mathcal{S}} \quad \wedge \quad \tilde{\mathcal{S}}^\star = \mathcal{S} \quad \textit{for some } \tilde{\mathcal{S}}$ [fetch$^\mu$·complete]

Proof.

$$\dfrac{}{\dfrac{\overset{\mathcal{D}_{11}}{\tilde{\mathcal{S}}^\star = \mathcal{S}} \quad \overset{\vdots}{\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu}}{(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu)^\star = (\mathcal{S};\mathcal{E}_\mu)}\,[\text{k}\cdot\text{env}^\star_2] \quad \wedge \quad (\mathcal{S};\mathcal{E}_\mu)(0) = \mathcal{S}\ ^{[\text{fetch}^\mu_1]} \quad \Rightarrow \quad (\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu)(0) = \tilde{\mathcal{S}}\ ^{[\text{i}\cdot\text{fetch}^\mu_1]} \quad \wedge \quad \overset{\mathcal{D}_{11}}{\tilde{\mathcal{S}}^\star = \mathcal{S}} \quad ^{[\text{fetch}^\mu\cdot\text{complete}]}}\,[\&1]$$

$$\dfrac{\overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu} \quad \wedge \quad \overset{\mathcal{D}_4}{\mathcal{E}_\mu(\alpha) = \mathcal{S}} \quad \Rightarrow \quad \overset{\mathcal{D}_2}{\tilde{\mathcal{E}}_\mu(\alpha) = \tilde{\mathcal{S}}} \quad \wedge \quad \overset{\mathcal{D}_3}{\tilde{\mathcal{S}}^\star = \mathcal{S}} \quad ^{[\text{fetch}^\mu\cdot\text{complete}]}}{\dfrac{\overset{\vdots}{\tilde{\mathcal{S}}'^\star = \mathcal{S}'} \quad \overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu}}{(\tilde{\mathcal{S}}';\tilde{\mathcal{E}}_\mu)^\star = (\mathcal{S}';\mathcal{E}_\mu)}\,[\text{k}\cdot\text{env}^\star_2] \quad \wedge \quad \dfrac{\overset{\mathcal{D}_4}{\mathcal{E}_\mu(\alpha) = \mathcal{S}}}{(\mathcal{S}';\mathcal{E}_\mu)(\alpha+1) = \mathcal{S}}\,[\text{fetch}^\mu_2] \quad \Rightarrow \quad \dfrac{\overset{\mathcal{D}_2}{\tilde{\mathcal{E}}_\mu(\alpha) = \tilde{\mathcal{S}}}}{(\tilde{\mathcal{S}}';\tilde{\mathcal{E}}_\mu)(\alpha+1) = \tilde{\mathcal{S}}}\,[\text{i}\cdot\text{fetch}^\mu_2] \quad \wedge \quad \overset{\mathcal{D}_3}{\tilde{\mathcal{S}}^\star = \mathcal{S}} \quad ^{[\text{fetch}^\mu\cdot\text{complete}]}}\,[\&2]$$

%mode $+\mathcal{D}_1 \quad \wedge \quad +\mathcal{D}_2 \quad \Rightarrow \quad -\mathcal{D}_3 \quad \wedge \quad -\mathcal{D}_4$ [fetch$^\mu$·complete]
%worlds $()\quad \mathcal{D}_1 \quad \wedge \quad \mathcal{D}_2 \quad \Rightarrow \quad \mathcal{D}_3 \quad \wedge \quad \mathcal{D}_4$ [fetch$^\mu$·complete]
%total $(\mathcal{D}_2)\quad \mathcal{D}_1 \quad \wedge \quad \mathcal{D}_2 \quad \Rightarrow \quad \mathcal{D}_3 \quad \wedge \quad \mathcal{D}_4$ [fetch$^\mu$·complete]

□

%theorem $\sigma_1 \rightsquigarrow \sigma_2 \quad \wedge \quad \tilde{\sigma}_1{}^\star = \sigma_1 \quad \Rightarrow \quad \tilde{\sigma}_1 \rightsquigarrow \tilde{\sigma}_2 \quad \wedge \quad \tilde{\sigma}_2{}^\star = \sigma_2 \quad \textit{for some } \tilde{\sigma}_2$ [completeness]

Proof.

$$\dfrac{\overset{\mathcal{D}_{212}}{\tilde{\mathcal{E}}^\star = \mathcal{E}} \quad \wedge \quad \overset{\mathcal{D}_4}{\mathcal{E}(g) = (u, \mathcal{E}', \mathcal{E}'_\mu)} \quad \Rightarrow \quad \overset{\mathcal{D}_{13}}{\tilde{\mathcal{E}}(g) = (t, n', \mathcal{I}', \mathcal{I}'_\mu, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_\mu)} \quad \wedge \quad \overset{\mathcal{D}_{31}}{(t, n', \mathcal{I}', \mathcal{I}'_\mu, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_\mu)^\star = (u, \mathcal{E}', \mathcal{E}'_\mu)} \quad ^{[\text{fetch}\cdot\text{complete}]}}{\dfrac{\overset{\mathcal{D}_4}{\mathcal{E}(g) = (u, \mathcal{E}', \mathcal{E}'_\mu)}}{\langle g, \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle u, \mathcal{E}', \mathcal{E}'_\mu, \mathcal{S}\rangle}\,[\text{k}\cdot\text{var}] \quad \wedge \quad \dfrac{\dfrac{\dfrac{\overset{\mathcal{D}_{11}}{n \dot{-} \mathcal{I}(l) = g}}{\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(l) = g}\,[\downarrow_1] \quad \overset{\mathcal{D}_{212}}{\tilde{\mathcal{E}}^\star = \mathcal{E}} \quad \overset{\vdots}{\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu}}{(l, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu)^\star = (g, \mathcal{E}, \mathcal{E}_\mu)}\,[\text{clos}^\star] \quad \overset{\mathcal{D}_{22}}{\tilde{\mathcal{S}}^\star = \mathcal{S}}}{\langle l, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle^\star = \langle g, \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}\rangle}\,[\text{state}^\star] \quad \Rightarrow \quad \dfrac{\overset{\mathcal{D}_{11}}{n \dot{-} \mathcal{I}(l) = g} \quad \overset{\mathcal{D}_{13}}{\tilde{\mathcal{E}}(g) = (t, n', \mathcal{I}', \mathcal{I}'_\mu, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_\mu)}}{\langle l, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t, n', \mathcal{I}', \mathcal{I}'_\mu, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_\mu, \tilde{\mathcal{S}}\rangle}\,[\text{i}\cdot\text{var}] \quad \wedge \quad \dfrac{\overset{\mathcal{D}_{31}}{(t, n', \mathcal{I}', \mathcal{I}'_\mu, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_\mu)^\star = (u, \mathcal{E}', \mathcal{E}'_\mu)} \quad \overset{\mathcal{D}_{22}}{\tilde{\mathcal{S}}^\star = \mathcal{S}}}{\langle t, n', \mathcal{I}', \mathcal{I}'_\mu, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_\mu, \tilde{\mathcal{S}}\rangle^\star = \langle u, \mathcal{E}', \mathcal{E}'_\mu, \mathcal{S}\rangle}\,[\text{state}^\star] \quad ^{[\text{completeness}]}}\,[\&1]$$

$$\dfrac{}{\langle (t'\,u'), \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t', \mathcal{E}, \mathcal{E}_\mu, (u', \mathcal{E}, \mathcal{E}_\mu) :: \mathcal{S}\rangle\ ^{[\text{k}\cdot\text{app}]} \quad \wedge \quad \dfrac{\dfrac{\dfrac{\overset{\mathcal{D}_{2111}}{\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t) = t'} \quad \overset{\mathcal{D}_{2112}}{\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(u) = u'}}{\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t\,u) = (t'\,u')}\,[\downarrow_2] \quad \overset{\mathcal{D}_{212}}{\tilde{\mathcal{E}}^\star = \mathcal{E}} \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu}}{((t\,u), n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu)^\star = ((t'\,u'), \mathcal{E}, \mathcal{E}_\mu)}\,[\text{clos}^\star] \quad \overset{\mathcal{D}_{23}}{\tilde{\mathcal{S}}^\star = \mathcal{S}}}{\langle (t\,u), n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle^\star = \langle (t'\,u'), \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}\rangle}\,[\text{state}^\star] \quad \Rightarrow \quad \langle (t\,u), n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, (u, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu) :: \tilde{\mathcal{S}}\rangle\ ^{[\text{i}\cdot\text{app}]} \quad \wedge \quad \dfrac{\dfrac{\overset{\mathcal{D}_{2111}}{\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t) = t'} \quad \overset{\mathcal{D}_{212}}{\tilde{\mathcal{E}}^\star = \mathcal{E}} \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu}}{(t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu)^\star = (t', \mathcal{E}, \mathcal{E}_\mu)}\,[\text{clos}^\star] \quad \dfrac{\dfrac{\overset{\mathcal{D}_{2112}}{\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(u) = u'} \quad \overset{\mathcal{D}_{212}}{\tilde{\mathcal{E}}^\star = \mathcal{E}} \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu}}{(u, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu)^\star = (u', \mathcal{E}, \mathcal{E}_\mu)}\,[\text{clos}^\star] \quad \overset{\mathcal{D}_{23}}{\tilde{\mathcal{S}}^\star = \mathcal{S}}}{((u, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu) :: \tilde{\mathcal{S}})^\star = (u', \mathcal{E}, \mathcal{E}_\mu) :: \mathcal{S}}\,[\text{stack}^\star_2]}{\langle t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, (u, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu) :: \tilde{\mathcal{S}}\rangle^\star = \langle t', \mathcal{E}, \mathcal{E}_\mu, (u', \mathcal{E}, \mathcal{E}_\mu) :: \mathcal{S}\rangle}\,[\text{state}^\star] \quad ^{[\text{completeness}]}}\,[\&2]$$

$$\dfrac{}{\langle \lambda t', \mathcal{E}, \mathcal{E}_\mu, c :: \mathcal{S}\rangle \rightsquigarrow \langle t', (c;\mathcal{E}), \mathcal{E}_\mu, \mathcal{S}\rangle\ ^{[\text{k}\cdot\text{abs}]} \quad \wedge \quad \dfrac{\dfrac{\dfrac{\overset{\mathcal{D}_{2111}}{\downarrow_{n+1}^{(n+1::\mathcal{I}),\mathcal{I}_\mu}(t) = t'}}{\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(\lambda t) = \lambda t'}\,[\downarrow_3] \quad \overset{\mathcal{D}_{212}}{\tilde{\mathcal{E}}^\star = \mathcal{E}} \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu}}{(\lambda t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu)^\star = (\lambda t', \mathcal{E}, \mathcal{E}_\mu)}\,[\text{clos}^\star] \quad \dfrac{\overset{\mathcal{D}_{221}}{\tilde{c}^\star = c} \quad \overset{\mathcal{D}_{222}}{\tilde{\mathcal{S}}^\star = \mathcal{S}}}{(\tilde{c} :: \tilde{\mathcal{S}})^\star = c :: \mathcal{S}}\,[\text{stack}^\star_2]}{\langle \lambda t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{c} :: \tilde{\mathcal{S}}\rangle^\star = \langle \lambda t', \mathcal{E}, \mathcal{E}_\mu, c :: \mathcal{S}\rangle}\,[\text{state}^\star] \quad \Rightarrow \quad \langle \lambda t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{c} :: \tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t, n+1, (n+1 :: \mathcal{I}), \mathcal{I}_\mu, (\tilde{c};\tilde{\mathcal{E}}), \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle\ ^{[\text{i}\cdot\text{abs}]} \quad \wedge \quad \dfrac{\dfrac{\overset{\mathcal{D}_{2111}}{\downarrow_{n+1}^{(n+1::\mathcal{I}),\mathcal{I}_\mu}(t) = t'} \quad \dfrac{\overset{\mathcal{D}_{221}}{\tilde{c}^\star = c} \quad \overset{\mathcal{D}_{212}}{\tilde{\mathcal{E}}^\star = \mathcal{E}}}{(\tilde{c};\tilde{\mathcal{E}})^\star = (c;\mathcal{E})}\,[\text{c}\cdot\text{env}^\star_2] \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu}}{(t, n+1, (n+1 :: \mathcal{I}), \mathcal{I}_\mu, (\tilde{c};\tilde{\mathcal{E}}), \tilde{\mathcal{E}}_\mu)^\star = (t', (c;\mathcal{E}), \mathcal{E}_\mu)}\,[\text{clos}^\star] \quad \overset{\mathcal{D}_{222}}{\tilde{\mathcal{S}}^\star = \mathcal{S}}}{\langle t, n+1, (n+1 :: \mathcal{I}), \mathcal{I}_\mu, (\tilde{c};\tilde{\mathcal{E}}), \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle^\star = \langle t', (c;\mathcal{E}), \mathcal{E}_\mu, \mathcal{S}\rangle}\,[\text{state}^\star] \quad ^{[\text{completeness}]}}\,[\&3]$$

$$\dfrac{}{\langle \textbf{catch}\, t', \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t', \mathcal{E}, (\mathcal{S};\mathcal{E}_\mu), \mathcal{S}\rangle\ ^{[\text{k}\cdot\text{catch}]} \quad \wedge \quad \dfrac{\dfrac{\dfrac{\overset{\mathcal{D}_{2111}}{\downarrow_n^{\mathcal{I},(\mathcal{I}::\mathcal{I}_\mu)}(t) = t'}}{\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(\textbf{get-context}\, t) = \textbf{catch}\, t'}\,[\downarrow_4] \quad \overset{\mathcal{D}_{212}}{\tilde{\mathcal{E}}^\star = \mathcal{E}} \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu}}{(\textbf{get-context}\, t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu)^\star = (\textbf{catch}\, t', \mathcal{E}, \mathcal{E}_\mu)}\,[\text{clos}^\star] \quad \overset{\mathcal{D}_{22}}{\tilde{\mathcal{S}}^\star = \mathcal{S}}}{\langle \textbf{get-context}\, t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle^\star = \langle \textbf{catch}\, t', \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}\rangle}\,[\text{state}^\star] \quad \Rightarrow \quad \langle \textbf{get-context}\, t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t, n, \mathcal{I}, (\mathcal{I} :: \mathcal{I}_\mu), \tilde{\mathcal{E}}, (\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu), \tilde{\mathcal{S}}\rangle\ ^{[\text{i}\cdot\text{catch}]} \quad \wedge \quad \dfrac{\dfrac{\overset{\mathcal{D}_{2111}}{\downarrow_n^{\mathcal{I},(\mathcal{I}::\mathcal{I}_\mu)}(t) = t'} \quad \overset{\mathcal{D}_{212}}{\tilde{\mathcal{E}}^\star = \mathcal{E}} \quad \dfrac{\overset{\mathcal{D}_{22}}{\tilde{\mathcal{S}}^\star = \mathcal{S}} \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu}}{(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu)^\star = (\mathcal{S};\mathcal{E}_\mu)}\,[\text{k}\cdot\text{env}^\star_2]}{(t, n, \mathcal{I}, (\mathcal{I} :: \mathcal{I}_\mu), \tilde{\mathcal{E}}, (\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu))^\star = (t', \mathcal{E}, (\mathcal{S};\mathcal{E}_\mu))}\,[\text{clos}^\star] \quad \overset{\mathcal{D}_{22}}{\tilde{\mathcal{S}}^\star = \mathcal{S}}}{\langle t, n, \mathcal{I}, (\mathcal{I} :: \mathcal{I}_\mu), \tilde{\mathcal{E}}, (\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu), \tilde{\mathcal{S}}\rangle^\star = \langle t', \mathcal{E}, (\mathcal{S};\mathcal{E}_\mu), \mathcal{S}\rangle}\,[\text{state}^\star] \quad ^{[\text{completeness}]}}\,[\&4]$$

$$\dfrac{\overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu} \quad \wedge \quad \overset{\mathcal{D}_4}{\mathcal{E}_\mu(\alpha) = \mathcal{S}'} \quad \Rightarrow \quad \overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}_\mu(\alpha) = \tilde{\mathcal{S}}'} \quad \wedge \quad \overset{\mathcal{D}_{32}}{\tilde{\mathcal{S}}'^\star = \mathcal{S}'} \quad ^{[\text{fetch}^\mu\cdot\text{complete}]}}{\dfrac{\overset{\mathcal{D}_4}{\mathcal{E}_\mu(\alpha) = \mathcal{S}'}}{\langle \textbf{throw}\, \alpha\, t', \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t', \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}'\rangle}\,[\text{k}\cdot\text{throw}] \quad \wedge \quad \dfrac{\dfrac{\dfrac{\overset{\mathcal{D}_{11}}{\mathcal{I}_\mu(\alpha) = \mathcal{I}'} \quad \overset{\mathcal{D}_{2112}}{\downarrow_n^{\mathcal{I}',\mathcal{I}_\mu}(t) = t'}}{\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(\textbf{set-context}\, \alpha\, t) = \textbf{throw}\, \alpha\, t'}\,[\downarrow_5] \quad \overset{\mathcal{D}_{212}}{\tilde{\mathcal{E}}^\star = \mathcal{E}} \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu}}{(\textbf{set-context}\, \alpha\, t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu)^\star = (\textbf{throw}\, \alpha\, t', \mathcal{E}, \mathcal{E}_\mu)}\,[\text{clos}^\star] \quad \overset{\vdots}{\tilde{\mathcal{S}}^\star = \mathcal{S}}}{\langle \textbf{set-context}\, \alpha\, t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle^\star = \langle \textbf{throw}\, \alpha\, t', \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}\rangle}\,[\text{state}^\star] \quad \Rightarrow \quad \dfrac{\overset{\mathcal{D}_{11}}{\mathcal{I}_\mu(\alpha) = \mathcal{I}'} \quad \overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}_\mu(\alpha) = \tilde{\mathcal{S}}'}}{\langle \textbf{set-context}\, \alpha\, t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t, n, \mathcal{I}', \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}'\rangle}\,[\text{i}\cdot\text{throw}] \quad \wedge \quad \dfrac{\dfrac{\overset{\mathcal{D}_{2112}}{\downarrow_n^{\mathcal{I}',\mathcal{I}_\mu}(t) = t'} \quad \overset{\mathcal{D}_{212}}{\tilde{\mathcal{E}}^\star = \mathcal{E}} \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu}}{(t, n, \mathcal{I}', \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu)^\star = (t', \mathcal{E}, \mathcal{E}_\mu)}\,[\text{clos}^\star] \quad \overset{\mathcal{D}_{32}}{\tilde{\mathcal{S}}'^\star = \mathcal{S}'}}{\langle t, n, \mathcal{I}', \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}'\rangle^\star = \langle t', \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}'\rangle}\,[\text{state}^\star] \quad ^{[\text{completeness}]}}\,[\&5]$$

%mode $+\mathcal{D}_1 \quad \wedge \quad +\mathcal{D}_2 \quad \Rightarrow \quad -\mathcal{D}_3 \quad \wedge \quad -\mathcal{D}_4$ [completeness]
%worlds $()\quad \mathcal{D}_1 \quad \wedge \quad \mathcal{D}_2 \quad \Rightarrow \quad \mathcal{D}_3 \quad \wedge \quad \mathcal{D}_4$ [completeness]
%total $(\mathcal{D}_1)\quad \mathcal{D}_1 \quad \wedge \quad \mathcal{D}_2 \quad \Rightarrow \quad \mathcal{D}_3 \quad \wedge \quad \mathcal{D}_4$ [completeness]

□