1 System T

1.1 Syntax

$bool$: **type**.
$unary$: **type**.
$term$: **type**.
$clos$: **type**.
env : **type**.
$stack$: **type**.
$state$: **type**.

Unary number n

0 : $unary$.
$n+1$: $unary$.

Boolean b

true : $bool$.
false : $bool$.

Term t

n : $term$.
$t_1\, t_2$: $term$.
$\lambda n.t$: $term$.

Closure c

$(t, \mathcal{E})$: $clos$.

Environment $\mathcal{E}$

$[]$: env.
$(\mathcal{E},\, n \leftarrow c)$: env.

Stack $\mathcal{S}$

$[]$: $stack$.
$c : \mathcal{S}$: $stack$.

State σ

$\langle t, \mathcal{E}, \mathcal{S} \rangle$: $state$.

1.2 Judgments

$n_1 = n_2 \rightsquigarrow b \quad : \quad \mathbf{type}.$
$\mathcal{E}(n) = c \quad : \quad \mathbf{type}.$
$\mathcal{E}(n_1)_{n_2} = c \quad : \quad \mathbf{type}.$
$\sigma_1 \to \sigma_2 \quad : \quad \mathbf{type}.$
$b?c_1 : c_2 \rightsquigarrow c \quad : \quad \mathbf{type}.$

1.3 Equality

$$\frac{}{0 = 0 \rightsquigarrow \mathbf{true}}\ [\text{Equal}_1]$$

$$\frac{}{0 = n + 1 \rightsquigarrow \mathbf{false}}\ [\text{Equal}_2]$$

$$\frac{}{n + 1 = 0 \rightsquigarrow \mathbf{false}}\ [\text{Equal}_3]$$

$$\frac{n = m \rightsquigarrow b}{n + 1 = m + 1 \rightsquigarrow b}\ [\text{Equal}_4]$$

%mode $+n = +m \rightsquigarrow -b$
%worlds $()\quad n = m \rightsquigarrow b$
%terminates $n \quad n = m \rightsquigarrow b$
%unique $+n = +m \rightsquigarrow -1b$

1.4 Conditional

$$\frac{}{\mathbf{true}?c_1 : c_2 \rightsquigarrow c_1}\ [\text{Cond\_1}]$$

$$\frac{}{\mathbf{false}?c_1 : c_2 \rightsquigarrow c_2}\ [\text{Cond\_2}]$$

%mode $+b? + c : +c' \rightsquigarrow -c''$
%worlds $()\quad b?c : c' \rightsquigarrow c''$
%terminates $b \quad b?c_1 : c_2 \rightsquigarrow c'$
%unique $+b? + c : +c' \rightsquigarrow -1c''$

1.5 Fetch

$$\frac{n = m \rightsquigarrow b \quad \mathcal{E}(m) = c' \quad b?c : c' \rightsquigarrow c''}{(\mathcal{E},\ n \leftarrow c)(m) = c''}\ [\text{Fetch\_1}]$$

%mode $+\mathcal{E}(+n) = -c$
%worlds $()\quad \mathcal{E}(n) = c$
%terminates $\mathcal{E} \quad \mathcal{E}(n) = c$
%unique $+\mathcal{E}(+n) = -1c$

1.6 Evaluation

$$\frac{\mathcal{E}(x) = (t, \mathcal{E}')}{\langle x, \mathcal{E}, \mathcal{S}\rangle \to \langle t, \mathcal{E}', \mathcal{S}\rangle}\ [\text{Eval\_Var}]$$

$$\frac{}{\langle (t_1\, t_2), \mathcal{E}, \mathcal{S}\rangle \to \langle t_1, \mathcal{E}, (t_2, \mathcal{E}) : \mathcal{S}\rangle}\ [\text{Eval\_App}]$$

$$\frac{}{\langle \lambda x.t, \mathcal{E}, c : \mathcal{S}\rangle \to \langle t, (\mathcal{E},\ x \leftarrow c), \mathcal{S}\rangle}\ [\text{Eval\_Abs}]$$

%mode $+\sigma_1 \to -\sigma_2$
%worlds $()\quad \sigma_1 \to \sigma_2$
%unique $+\sigma_1 \to -1\sigma_2$