1 System T

1.1 Syntax

$term$: **type**.
$clos$: **type**.
env : **type**.
$stack$: **type**.
$state$: **type**.
int : **type**.
$bool$: **type**.
$frame$: **type**.

Term t

x : $term$.
m : $term$.
$t_1\, t_2$: $term$.
$\lambda x.t$: $term$.
$\mathbf{succ}(t)$: $term$.
$\mathbf{pred}(t)$: $term$.
$\mathbf{rec}(t_1, t_2, t_3)$: $term$.
$\mathbf{let}\ x = t_1\ \mathbf{in}\ t_2$: $term$.

Integer m

$\mathbf{Z}$: int.
$\mathbf{S}(m)$: int.

Boolean b

$\boldsymbol{T}$: $bool$.
$\boldsymbol{F}$: $bool$.

Closure c

$(t; \mathcal{E})$: $clos$.

Environment $\mathcal{E}$

$[]$: env.
$(\mathcal{E},\ x \leftarrow c)$: env.

Frame f

$([]\ c)$: $frame$.
$(c\ [])$: $frame$.
$\mathbf{rec}([], c_2, c_3)$: $frame$.
$\mathbf{rec}(c_1, [], c_3)$: $frame$.
$\mathbf{rec}(c_1, c_2, [])$: $frame$.
$\mathbf{succ}([])$: $frame$.

Stack $\mathcal{S}$

$[]$: $stack$.

$f : \mathcal{S} \quad : \quad \mathit{stack}.$

State $\boldsymbol{\sigma}$

$\langle t, \mathcal{E}, \mathcal{S} \rangle \quad : \quad \mathit{state}.$

1.2 Judgments

$\mathcal{E}(x) = c \quad : \quad \mathbf{type}.$
$\sigma 1 \rightarrow \sigma 2 \quad : \quad \mathbf{type}.$
$\sigma 1 \rightsquigarrow c_2 \quad : \quad \mathbf{type}.$
$t\ \mathbf{value} \quad : \quad \mathbf{type}.$

1.3 Fetch

$$\frac{}{(\mathcal{E},\ x \leftarrow c)(x) = c}\ \text{[Fetch1]}$$

$$\frac{x \neq x' \quad \mathcal{E}(x) = c}{(\mathcal{E},\ x' \leftarrow c')(x) = c}\ \text{[Fetch2]}$$

%mode $\quad +\mathcal{E}(+x) = -c$
%worlds $\quad () \quad \mathcal{E}(x) = c$
%terminates $\quad \mathcal{E} \quad \mathcal{E}(x) = c$

Remark. Twelf cannot check the following property:
%unique $\quad +\mathcal{E}(+x) = -1c$

1.4 Value

$$\frac{}{m\ \mathbf{value}}\ \text{[V\_Cst]}$$

$$\frac{}{x\ \mathbf{value}}\ \text{[V\_Var]}$$

$$\frac{}{\lambda x.t\ \mathbf{value}}\ \text{[V\_Abs]}$$

1.5 Evaluation

$$\frac{\mathcal{E}(x) = (t; \mathcal{E}')}{\langle x, \mathcal{E}, \mathcal{S} \rangle \rightarrow \langle t, \mathcal{E}', \mathcal{S} \rangle}\ \text{[E\_Var]}$$

$$\frac{}{\langle t\,t', \mathcal{E}, \mathcal{S} \rangle \rightarrow \langle t', \mathcal{E}, ((t; \mathcal{E})\ []) : \mathcal{S} \rangle}\ \text{[E\_App1]}$$

$$\frac{w\ \mathbf{value}}{\langle w, \mathcal{E}, (((t; \mathcal{E}')\ [])) : \mathcal{S} \rangle \rightarrow \langle t, \mathcal{E}', ([]\ (w; \mathcal{E})) : \mathcal{S} \rangle}\ \text{[E\_App2]}$$

$$\frac{}{\langle \lambda x.t, \mathcal{E}, ([]\ c) : \mathcal{S} \rangle \rightarrow \langle t, (\mathcal{E},\ x \leftarrow c), \mathcal{S} \rangle}\ \text{[E\_Abs]}$$

$$\frac{}{\langle \mathbf{let}\ x = t_1\ \mathbf{in}\ t_2, \mathcal{E}, \mathcal{S} \rangle \rightarrow \langle t_1, \mathcal{E}, ((\lambda x.t_2; \mathcal{E})\ []) : \mathcal{S} \rangle}\ \text{[E\_Let]}$$

$$\frac{}{\langle \mathbf{succ}(t), \mathcal{E}, \mathcal{S} \rangle \rightarrow \langle t, \mathcal{E}, \mathbf{succ}([]) : \mathcal{S} \rangle}\ \text{[E\_succ1]}$$

$$\frac{}{\langle m, \mathcal{E}, \mathbf{succ}([]) : \mathcal{S} \rangle \rightarrow \langle \mathbf{S}(m), \mathcal{E}, \mathcal{S} \rangle}\ \text{[E\_succ2]}$$

$$\frac{}{\langle \mathbf{rec}(t_1, t_2, t_3), \mathcal{E}, \mathcal{S} \rangle \rightarrow \langle t_3, \mathcal{E}, \mathbf{rec}((t_1; \mathcal{E}), (t_2; \mathcal{E}), []) : \mathcal{S} \rangle}\ \text{[E\_rec3]}$$

$$\frac{w\ \mathbf{value}}{\langle w, \mathcal{E}, \mathbf{rec}((t_1;\mathcal{E}_1), (t_2;\mathcal{E}_2)\ , []) : \mathcal{S}\rangle \rightarrow \langle t_2, \mathcal{E}_2, \mathbf{rec}((t_1;\mathcal{E}), [], (w;\mathcal{E})) : \mathcal{S}\rangle}\ [\text{E\_rec4}]$$

$$\frac{w\ \mathbf{value} \quad w_3\ \mathbf{value}}{\langle w, \mathcal{E}, \mathbf{rec}((t_1;\mathcal{E}_1), [], (w_3;\mathcal{E}_3)) : \mathcal{S}\rangle \rightarrow \langle t_1, \mathcal{E}_1, \mathbf{rec}([], (w;\mathcal{E}), (w_3;\mathcal{E}_3)) : \mathcal{S}\rangle}\ [\text{E\_rec5}]$$

$$\frac{w_2\ \mathbf{value} \quad w_3\ \mathbf{value}}{\langle \mathbf{Z}, \mathcal{E}, \mathbf{rec}([], (w_2;\mathcal{E}_2), (w_3;\mathcal{E}_3)) : \mathcal{S}\rangle \rightarrow \langle w_2, \mathcal{E}_2, \mathcal{S}\rangle}\ [\text{E\_rec1}]$$

$$\frac{w_2\ \mathbf{value} \quad w_3\ \mathbf{value}}{\langle \mathbf{S}(m), \mathcal{E}, \mathbf{rec}([], (w_2;\mathcal{E}_2), (w_3;\mathcal{E}_3)) : \mathcal{S}\rangle \rightarrow \langle m, \mathcal{E}, \mathbf{rec}([], (w_2;\mathcal{E}_2), (w_3;\mathcal{E}_3)) : (((w_3\, m);\mathcal{E}_3)\ []) : \mathcal{S}\rangle}\ [\text{E\_rec2}]$$

%mode $+t$ **value**
%worlds $()$ t **value**
%mode $+\sigma_1 \rightarrow -\sigma_2$
%worlds $()$ $\sigma_1 \rightarrow \sigma_2$

Remark. Twelf cannot check the following property:
%unique $+\sigma_1 \rightarrow -1\sigma_2$

1.6 Full evaluation

$$\frac{m\ \mathbf{value}}{\langle m, \mathcal{E}, []\rangle \rightsquigarrow (m;\mathcal{E})}\ [\text{Eval\_1}]$$

$$\frac{\sigma_1 \rightarrow \sigma_2 \quad \sigma_2 \rightsquigarrow c}{\sigma_1 \rightsquigarrow c}\ [\text{Eval\_2}]$$

%solve $\langle \mathbf{succ}(\mathbf{Z}), [], []\rangle \rightsquigarrow \sqcup$
%solve $\langle \mathbf{let}\ \mathsf{g}{=}\lambda\mathsf{x}.\mathbf{succ}(\mathsf{x})\ \mathbf{in}\ (\mathsf{g}\,\mathbf{Z}), [], []\rangle \rightsquigarrow \sqcup$