1 Second simulation (completeness)

1.1 Syntax

1.1.1 Index, indices and tables

%datatype *index*
%name *index* n α

$$\begin{array}{lcl} n & ::= & 0 \\ & \mid & n+1 \end{array}$$

%datatype *vector*
%name *vector* $\mathcal{I}$

$$\begin{array}{lcl} \mathcal{I} & ::= & [] \\ & \mid & n{::}\mathcal{I} \end{array}$$

%datatype *table*
%name *table* $\mathcal{I}_\mu$

$$\begin{array}{lcl} \mathcal{I}_\mu & ::= & [] \\ & \mid & \mathcal{I}::\mathcal{I}_\mu \end{array}$$

1.1.2 Term

%datatype *term*
%name *term* t

$$\begin{array}{lcl} t & ::= & n \\ & \mid & t_1\,t_2 \\ & \mid & \lambda t \\ & \mid & \textbf{get-context}\,t \\ & \mid & \textbf{set-context}\,\alpha\,t \end{array}$$

1.2 Subtraction

%judgment $n_1 \dot{-} n_2 = n_3$

$$n_1 \dot{-} 0 = n_1\,{}^{[\text{minus}_1]}$$
$$(n_1+1) \dot{-} (n_2+1) = n_3\,{}^{[\text{minus}_2]} \quad \textit{when} \quad n_1 \dot{-} n_2 = n_3$$

%mode $+n_1 \dot{-} +n_2 = -n_3$
%worlds $()$ $n_1 \dot{-} n_2 = n_3$
%terminates (n_1) $n_1 \dot{-} n_2 = n_3$
%unique $+n_1 \dot{-} +n_2 = -1n_3$

%lemma $\forall n \cdot n \dot{-} n = 0$ $^{[\text{minus·equals}]}$

Proof.

$$\dfrac{}{0\colon index \cdot \quad 0 \dot{-} 0 = 0\,{}^{[\text{minus}_1]} \qquad {}^{[\text{minus·equals}]}}\ [\&1]$$

$$\dfrac{n\colon index \cdot \quad \begin{array}{c}\mathcal{D}\\ n \dot{-} n = 0\end{array} \qquad {}^{[\text{minus·equals}]}}{n+1\colon index \cdot \quad \dfrac{\begin{array}{c}\mathcal{D}\\ n \dot{-} n = 0\end{array}}{(n+1) \dot{-} (n+1) = 0}\,[\text{minus}_2] \qquad {}^{[\text{minus·equals}]}}\ [\&2]$$

%mode $+n \cdot -\mathcal{D}$ $^{[\text{minus·equals}]}$
%worlds $()$ $n \cdot \mathcal{D}$ $^{[\text{minus·equals}]}$
%total n $n \cdot \mathcal{D}$ $^{[\text{minus·equals}]}$

1.3 Equality

%judgment $n_1 = n_2$

$$n = n\,{}^{[\text{refl}]}$$

%mode $+n = +m$
%worlds $()$ $n = m$

%lemma $n_1 \dot{-} n_2 = n_3 \quad \wedge \quad n_1 \dot{-} n_2 = n_3' \quad \Rightarrow \quad n_3 = n_3'$ $^{[\text{minus·unique}]}$

Proof.

$$\dfrac{}{\begin{array}{c}\vdots\\ n_1 \dot{-} n_2 = n_3\end{array} \quad \wedge \quad \begin{array}{c}\vdots\\ n_1 \dot{-} n_2 = n_3\end{array} \quad \Rightarrow \quad n_3 = n_3\,{}^{[\text{refl}]} \qquad {}^{[\text{minus·unique}]}}\ [\&1]$$

%mode $+\mathcal{D}_1 \quad \wedge \quad +\mathcal{D}_2 \quad \Rightarrow \quad -\mathcal{D}_3$ $^{[\text{minus·unique}]}$
%worlds $()$ $\mathcal{D}_1 \quad \wedge \quad \mathcal{D}_2 \quad \Rightarrow \quad \mathcal{D}_3$ $^{[\text{minus·unique}]}$
%terminates $\{\}$ $\mathcal{D}_1 \quad \wedge \quad \mathcal{D}_2 \quad \Rightarrow \quad \mathcal{D}_3$ $^{[\text{minus·unique}]}$
%total $\{\}$ $\mathcal{D}_1 \quad \wedge \quad \mathcal{D}_2 \quad \Rightarrow \quad \mathcal{D}_3$ $^{[\text{minus·unique}]}$

1.3.1 Fetch (indices)

%judgment $\mathcal{I}(n_1) = n_2$

$$(n{::}\mathcal{I})(0) = n\,{}^{[\text{fetch}_1^{\mathcal{I}}]}$$
$$(n{::}\mathcal{I})(n_1+1) = n_2\,{}^{[\text{fetch}_2^{\mathcal{I}}]} \quad \textit{when} \quad \mathcal{I}(n_1) = n_2$$

%mode $+\mathcal{I}(+n_1) = -n_2$
%worlds $()$ $\mathcal{I}(n_1) = n_2$
%terminates n_1 $\mathcal{I}(n_1) = n_2$
%unique $+\mathcal{I}(+n_1) = -1n_2$

1.3.2 Fetch (table)

%judgment $\mathcal{I}_\mu(n) = \mathcal{I}$

$$(\mathcal{I}::\mathcal{I}_\mu)(0) = \mathcal{I}^{\left[\text{fetch}_1^{\mathcal{I}_\mu}\right]}$$
$$(\mathcal{I}'::\mathcal{I}_\mu)(\alpha+1) = \mathcal{I}^{\left[\text{fetch}_2^{\mathcal{I}_\mu}\right]} \quad \textit{when} \quad \mathcal{I}_\mu(\alpha) = \mathcal{I}$$

%mode $+\mathcal{I}_\mu(+\alpha) = -\mathcal{I}$
%worlds $()$ $\mathcal{I}_\mu(\alpha) = \mathcal{I}$
%terminates α $\mathcal{I}_\mu(\alpha) = \mathcal{I}$
%unique $+\mathcal{I}_\mu(+\alpha) = -1\mathcal{I}$

1.3.3 Compute

%judgment $n_1 \dot{-} \mathcal{I}(n_2) = n_3$

$$n \dot{-} \mathcal{I}(l) = g\,{}^{[\text{compute}_1]} \quad \textit{when} \quad \mathcal{I}(l) = k, \quad n \dot{-} k = g$$

%mode $+n_1 \dot{-} +\mathcal{I}(+n_2) = -n_3$
%worlds $()$ $n_1 \dot{-} \mathcal{I}(n_2) = n_3$
%terminates $\{\}$ $n_1 \dot{-} \mathcal{I}(n_2) = n_3$
%unique $+n_1 \dot{-} +\mathcal{I}(+n_2) = -1n_3$

1.3.4 Closure, environment and stack

%datatype *clos* **%name** *clos* c
%datatype *l-env* **%name** *l-env* $\mathcal{L}$
%datatype *l-table* **%name** *l-table* $\mathcal{L}_\mu$
%datatype *k-env* **%name** *k-env* $\mathcal{E}_\mu$
%datatype *stack* **%name** *stack* $\mathcal{S}$

$$\begin{array}{lcl} c & ::= & (t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu) \\[6pt] \mathcal{L} & ::= & () \\ & \mid & (c; \mathcal{L}) \\[6pt] \mathcal{L}_\mu & ::= & () \\ & \mid & \mathcal{L}\colon \mathcal{L}_\mu \\[6pt] \mathcal{E}_\mu & ::= & () \\ & \mid & (\mathcal{S}; \mathcal{E}_\mu) \\[6pt] \mathcal{S} & ::= & [] \\ & \mid & c::\mathcal{S} \end{array}$$

%datatype *state*
%name *state* σ

$$\sigma \quad ::= \quad \langle t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}\rangle$$

1.4 Judgments

1.4.1 Fetch a local closure

%judgment $\mathcal{L}(n) = c$

$$(c; \mathcal{L})(0) = c\,{}^{[\text{fetch}_1]}$$
$$(c'; \mathcal{L})(n+1) = c\,{}^{[\text{fetch}_2]} \quad \textit{when} \quad \mathcal{L}(n) = c$$

%mode $+\mathcal{L}(+n) = -c$
%worlds $()$ $\mathcal{L}(n) = c$
%terminates $\mathcal{L}$ $\mathcal{L}(n) = c$
%unique $+\mathcal{L}(+n) = -1c$

1.4.2 Fetch a local environment

%judgment $\mathcal{L}_\mu(n) = \mathcal{L}$

$$(\mathcal{L}\colon \mathcal{L}_\mu)(0) = \mathcal{L}\,{}^{[\text{l·fetch}_1]}$$
$$(\mathcal{L}'\colon \mathcal{L}_\mu)(n+1) = \mathcal{L}\,{}^{[\text{l·fetch}_2]} \quad \textit{when} \quad \mathcal{L}_\mu(n) = \mathcal{L}$$

%mode $+\mathcal{L}_\mu(+n) = -\mathcal{L}$
%worlds $()$ $\mathcal{L}_\mu(n) = \mathcal{L}$
%terminates $\mathcal{L}_\mu$ $\mathcal{L}_\mu(n) = \mathcal{L}$
%unique $+\mathcal{L}_\mu(+n) = -1\mathcal{L}$

1.4.3 Fetch a stack

%judgment $\mathcal{E}_\mu(n) = \mathcal{S}$

$$(\mathcal{S}; \mathcal{E}_\mu)(0) = \mathcal{S}\,{}^{[\text{fetch}_1^{\mu}]}$$
$$(\mathcal{S}'; \mathcal{E}_\mu)(n+1) = \mathcal{S}\,{}^{[\text{fetch}_2^{\mu}]} \quad \textit{when} \quad \mathcal{E}_\mu(n) = \mathcal{S}$$

%mode $+\mathcal{E}_\mu(+n) = -\mathcal{S}$
%worlds $()$ $\mathcal{E}_\mu(n) = \mathcal{S}$
%terminates $\mathcal{E}_\mu$ $\mathcal{E}_\mu(n) = \mathcal{S}$
%unique $+\mathcal{E}_\mu(+n) = -1\mathcal{S}$

1.4.4 Evaluation rules

%judgment $\sigma_1 \rightsquigarrow \sigma_2$

$$\langle k, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t, \mathcal{L}', \mathcal{L}_\mu', \mathcal{E}_\mu', \mathcal{S}\rangle\,{}^{[\text{k·var}]} \quad \textit{when} \quad \mathcal{L}(k) = (t, \mathcal{L}', \mathcal{L}_\mu', \mathcal{E}_\mu')$$
$$\langle (t\,u), \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, (u, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu)::\mathcal{S}\rangle\,{}^{[\text{k·app}]}$$
$$\langle \lambda t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, c::\mathcal{S}\rangle \rightsquigarrow \langle t, (c; \mathcal{L}), \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}\rangle\,{}^{[\text{k·abs}]}$$
$$\langle \textbf{get-context}\,t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t, \mathcal{L}, (\mathcal{L}\colon \mathcal{L}_\mu), (\mathcal{S}; \mathcal{E}_\mu), \mathcal{S}\rangle\,{}^{[\text{k·catch}]}$$
$$\langle \textbf{set-context}\,\alpha\,t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t, \mathcal{L}', \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}'\rangle\,{}^{[\text{k·throw}]} \quad \textit{when} \quad \mathcal{L}_\mu(\alpha) = \mathcal{L}', \quad \mathcal{E}_\mu(\alpha) = \mathcal{S}'$$

%mode $+\sigma_1 \rightsquigarrow -\sigma_2$
%worlds $()$ $\sigma_1 \rightsquigarrow \sigma_2$
%unique $+\sigma_1 \rightsquigarrow -1\sigma_2$

2 Abstract machine for safe $\boldsymbol{\lambda_{ct}}$-terms

2.0.5 Syntax

%datatype *clos*
%datatype *c-env*
%datatype *k-env*
%datatype *stack*
%name *clos* $\tilde{c}$
%name *c-env* $\tilde{\mathcal{E}}$
%name *k-env* $\tilde{\mathcal{E}}_\mu$
%name *stack* $\tilde{\mathcal{S}}$

$$\tilde{c} \quad ::= \quad \left(t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu\right)$$

$$\begin{array}{lll} \tilde{\mathcal{E}} & ::= & () \\ & \mid & (\tilde{c};\tilde{\mathcal{E}}) \end{array}$$

$$\begin{array}{lll} \tilde{\mathcal{E}}_\mu & ::= & () \\ & \mid & (\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu) \end{array}$$

$$\begin{array}{lll} \tilde{\mathcal{S}} & ::= & [] \\ & \mid & \tilde{c} :: \tilde{\mathcal{S}} \end{array}$$

%datatype *state*
%name *state* $\tilde{\sigma}$

$$\tilde{\sigma} \quad ::= \quad \langle t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}} \rangle$$

2.0.6 Fetch a closure

%judgment $\tilde{\mathcal{E}}(n) = \tilde{c}$

$$(\tilde{c};\tilde{\mathcal{E}})(0) = \tilde{c}\ ^{[\text{i·fetch}_1]}$$
$$(\tilde{c}';\tilde{\mathcal{E}})(n+1) = \tilde{c}\ ^{[\text{i·fetch}_2]} \quad \textit{when} \quad \tilde{\mathcal{E}}(n) = \tilde{c}$$

%mode $+\tilde{\mathcal{E}}(+n) = -\tilde{c}$
%worlds $()$ $\tilde{\mathcal{E}}(n) = \tilde{c}$
%terminates $\tilde{\mathcal{E}}$ $\tilde{\mathcal{E}}(n) = \tilde{c}$
%unique $+\tilde{\mathcal{E}}(+n) = -1\tilde{c}$

2.0.7 Compute

%judgment $\tilde{\mathcal{E}}(n_1 \dot{-} n_2) = \tilde{c}$

$$\tilde{\mathcal{E}}(n \dot{-} k) = \tilde{c}\ ^{[\text{i·compute}_1]} \quad \textit{when} \quad n \dot{-} k = g, \quad \tilde{\mathcal{E}}(g) = \tilde{c}$$

%mode $+\tilde{\mathcal{E}}(+n \dot{-} +k) = -\tilde{c}$
%worlds $()$ $\tilde{\mathcal{E}}(n \dot{-} k) = \tilde{c}$
%terminates $\{\}$ $\tilde{\mathcal{E}}(n \dot{-} k) = \tilde{c}$
%unique $+\tilde{\mathcal{E}}(+n \dot{-} +k) = -1\tilde{c}$

2.0.8 Fetch a stack

%judgment $\tilde{\mathcal{E}}_\mu(n) = \tilde{\mathcal{S}}$

$$(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu)(0) = \tilde{\mathcal{S}}\ ^{[\text{i·fetch}^\mu_1]}$$
$$(\tilde{\mathcal{S}}';\tilde{\mathcal{E}}_\mu)(n+1) = \tilde{\mathcal{S}}\ ^{[\text{i·fetch}^\mu_2]} \quad \textit{when} \quad \tilde{\mathcal{E}}_\mu(n) = \tilde{\mathcal{S}}$$

%mode $+\tilde{\mathcal{E}}_\mu(+n) = -\tilde{\mathcal{S}}$
%worlds $()$ $\tilde{\mathcal{E}}_\mu(n) = \tilde{\mathcal{S}}$
%terminates $\tilde{\mathcal{E}}_\mu$ $\tilde{\mathcal{E}}_\mu(n) = \tilde{\mathcal{S}}$
%unique $+\tilde{\mathcal{E}}_\mu(+n) = -1\tilde{\mathcal{S}}$

2.1 Evaluation rules

%judgment $\tilde{\sigma}_1 \rightsquigarrow \tilde{\sigma}_2$

$$\langle l, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}} \rangle \rightsquigarrow \langle t, n', \mathcal{I}', \mathcal{I}'_\mu, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_\mu, \tilde{\mathcal{S}} \rangle\ ^{[\text{i·var}]}$$
$$\qquad \textit{when} \quad n \dot{-} \mathcal{I}(l) = g, \quad \tilde{\mathcal{E}}(g) = \left(t, n', \mathcal{I}', \mathcal{I}'_\mu, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_\mu\right)$$

$$\langle (t\,u), n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}} \rangle \rightsquigarrow \langle t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \left(u, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu\right) :: \tilde{\mathcal{S}} \rangle\ ^{[\text{i·app}]}$$

$$\langle \lambda t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{c} :: \tilde{\mathcal{S}} \rangle \rightsquigarrow \langle t, n+1, (n+1 :: \mathcal{I}), \mathcal{I}_\mu, (\tilde{c};\tilde{\mathcal{E}}), \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}} \rangle\ ^{[\text{i·abs}]}$$

$$\langle \textbf{get-context}\ t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}} \rangle \rightsquigarrow \langle t, n, \mathcal{I}, (\mathcal{I} :: \mathcal{I}_\mu), \tilde{\mathcal{E}}, (\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu), \tilde{\mathcal{S}} \rangle\ ^{[\text{i·catch}]}$$

$$\langle \textbf{set-context}\ \alpha\, t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}} \rangle \rightsquigarrow \langle t, n, \mathcal{I}', \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}' \rangle\ ^{[\text{i·throw}]}$$
$$\qquad \textit{when} \quad \mathcal{I}_\mu(\alpha) = \mathcal{I}', \quad \tilde{\mathcal{E}}_\mu(\alpha) = \tilde{\mathcal{S}}'$$

%mode $+\sigma_1 \rightsquigarrow -\sigma_2$
%worlds $()$ $\sigma_1 \rightsquigarrow \sigma_2$
%unique $+\sigma_1 \rightsquigarrow -1\sigma_2$

3 Translation

%judgment $\tilde{c}^\diamond = c$
%judgment $\tilde{\mathcal{S}}^\diamond = \mathcal{S}$
%judgment $\tilde{\mathcal{E}}_\mu{}^\diamond = \mathcal{E}_\mu$
%judgment $\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}$
%judgment $\textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ \mathcal{I}_\mu = \mathcal{L}_\mu$

$$\left(t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu\right)^\diamond = (t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu)\ ^{[\text{clos}^\diamond]} \quad \textit{when} \quad \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}, \quad \textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ \mathcal{I}_\mu = \mathcal{L}_\mu, \quad \tilde{\mathcal{E}}_\mu{}^\diamond = \mathcal{E}_\mu$$

$$[]^\diamond = []\ ^{[\text{stack}^\diamond_1]}$$
$$\left(\tilde{c} :: \tilde{\mathcal{S}}\right)^\diamond = c :: \mathcal{S}\ ^{[\text{stack}^\diamond_2]} \quad \textit{when} \quad \tilde{c}^\diamond = c, \quad \tilde{\mathcal{S}}^\diamond = \mathcal{S}$$

$$()^\diamond = ()\ ^{[\text{k·env}^\diamond_1]}$$
$$(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu)^\diamond = (\mathcal{S};\mathcal{E}_\mu)\ ^{[\text{k·env}^\diamond_2]} \quad \textit{when} \quad \tilde{\mathcal{S}}^\diamond = \mathcal{S}, \quad \tilde{\mathcal{E}}_\mu{}^\diamond = \mathcal{E}_\mu$$

$$\textit{flatten}\ n\ \tilde{\mathcal{E}}\ [] = ()\ ^{[\text{flatten}_1]}$$
$$\textit{flatten}\ n\ \tilde{\mathcal{E}}\ (k :: \mathcal{I}) = (c;\mathcal{L})\ ^{[\text{flatten}_2]} \quad \textit{when} \quad \tilde{\mathcal{E}}(n \dot{-} k) = \tilde{c}, \quad \tilde{c}^\diamond = c, \quad \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}$$

$$\textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ [] = ()\ ^{[\text{map}_1]}$$
$$\textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ (\mathcal{I} :: \mathcal{I}_\mu) = \mathcal{L} : \mathcal{L}_\mu\ ^{[\text{map}_2]} \quad \textit{when} \quad \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}, \quad \textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ \mathcal{I}_\mu = \mathcal{L}_\mu$$

%mode
$+\tilde{c}^\diamond = -c$
$+\tilde{\mathcal{S}}^\diamond = -\mathcal{S}$
$+\tilde{\mathcal{E}}_\mu{}^\diamond = -\mathcal{E}_\mu$
$\textit{flatten}\ +n\ +\tilde{\mathcal{E}}\ +\mathcal{I} = -\mathcal{L}$
$\textit{map}\ \left(\textit{flatten}\ +n\ +\tilde{\mathcal{E}}\right)\ +\mathcal{I}_\mu = -\mathcal{L}_\mu$

%worlds $()$
$\tilde{c}^\diamond = c$
$\tilde{\mathcal{S}}^\diamond = \mathcal{S}$
$\tilde{\mathcal{E}}_\mu{}^\diamond = \mathcal{E}_\mu$
$\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}$
$\textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ \mathcal{I}_\mu = \mathcal{L}_\mu$

Remark. To do.

%terminates $\left(\tilde{c}\ \tilde{\mathcal{S}}\ \tilde{\mathcal{E}}_\mu\ \mathcal{I}\ \mathcal{I}_\mu\right)$
$\tilde{c}^\diamond = c$
$\tilde{\mathcal{S}}^\diamond = \mathcal{S}$
$\tilde{\mathcal{E}}_\mu{}^\diamond = \mathcal{E}_\mu$
$\textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}$
$\textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ \mathcal{I}_\mu = \mathcal{L}_\mu$

%unique
$+\tilde{c}^\diamond = -1c$
$+\tilde{\mathcal{S}}^\diamond = -1\mathcal{S}$
$+\tilde{\mathcal{E}}_\mu{}^\diamond = -1\mathcal{E}_\mu$
$\textit{flatten}\ +n\ +\tilde{\mathcal{E}}\ +\mathcal{I} = -1\mathcal{L}$
$\textit{map}\ \left(\textit{flatten}\ +n\ +\tilde{\mathcal{E}}\right)\ +\mathcal{I}_\mu = -1\mathcal{L}_\mu$

%judgment $\tilde{\sigma}^\diamond = \sigma$

$$\langle t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}} \rangle^\diamond = \langle t', \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S} \rangle\ ^{[\text{state}^\diamond]} \quad \textit{when} \quad \left(t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu\right)^\diamond = (t', \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu), \quad \tilde{\mathcal{S}}^\diamond = \mathcal{S}$$

%mode $+\tilde{\sigma}^\diamond = -\sigma$
%worlds $()$ $\tilde{\sigma}^\diamond = \sigma$
%unique $+\tilde{\sigma}^\diamond = -1\sigma$

4 Completeness

%lemma $n \dot{-} k = g \quad \Rightarrow \quad (n+1) \dot{-} k = g+1 \quad ^{[\text{minus·succ}]}$

Proof.

$$\dfrac{}{n \dot{-} 0 = n\ ^{[\text{minus}_1]} \quad \Rightarrow \quad (n+1) \dot{-} 0 = (n+1)\ ^{[\text{minus}_1]} \quad ^{[\text{minus·succ}]}}\ [\&1]$$

$$\dfrac{\begin{array}{c}\mathcal{D}_1\\ n \dot{-} k = g\end{array} \quad \Rightarrow \quad \begin{array}{c}\mathcal{D}_2\\ (n+1) \dot{-} k = g+1\end{array} \quad ^{[\text{minus·succ}]}}{\dfrac{\begin{array}{c}\mathcal{D}_1\\ n \dot{-} k = g\end{array}}{(n+1) \dot{-} (k+1) = g}\,[\text{minus}_2] \quad \Rightarrow \quad \dfrac{\begin{array}{c}\mathcal{D}_2\\ (n+1) \dot{-} k = g+1\end{array}}{(n+1+1) \dot{-} (k+1) = g+1}\,[\text{minus}_2] \quad ^{[\text{minus·succ}]}}\ [\&1]$$

%mode $+\mathcal{D}_1 \quad \Rightarrow \quad -\mathcal{D}_2 \quad ^{[\text{minus·succ}]}$
%worlds $() \quad \mathcal{D}_1 \quad \Rightarrow \quad \mathcal{D}_2 \quad ^{[\text{minus·succ}]}$
%total $(\mathcal{D}_1) \quad \mathcal{D}_1 \quad \Rightarrow \quad \mathcal{D}_2 \quad ^{[\text{minus·succ}]}$

$\square$

%lemma $\forall \tilde{c}' \cdot \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L} \quad \Rightarrow \quad \textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\ \mathcal{I} = \mathcal{L} \quad ^{[\text{weaken·flatten}]}$

Proof.

$$\dfrac{}{\tilde{c}' : \textit{clos} \cdot \quad \textit{flatten}\ n\ \tilde{\mathcal{E}}\ [] = ()\ ^{[\text{flatten}_1]} \quad \Rightarrow \quad \textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\ [] = ()\ ^{[\text{flatten}_1]} \quad ^{[\text{weaken·flatten}]}}\ [\&1]$$

$$\dfrac{\begin{array}{c} \tilde{c}' : \textit{clos} \cdot \quad \begin{array}{c}\mathcal{D}_{23}\\ \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}\end{array} \quad \Rightarrow \quad \begin{array}{c}\mathcal{D}_3\\ \textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\ \mathcal{I} = \mathcal{L}\end{array} \quad ^{[\text{weaken·flatten}]} \\ \begin{array}{c}\mathcal{D}_{211}\\ n \dot{-} k = g\end{array} \quad \Rightarrow \quad \begin{array}{c}\mathcal{D}_4\\ (n+1) \dot{-} k = g+1\end{array} \quad ^{[\text{minus·succ}]} \end{array}}{\tilde{c}' : \textit{clos} \cdot \quad \dfrac{\dfrac{\begin{array}{c}\mathcal{D}_{211}\\ n \dot{-} k = g\end{array} \quad \begin{array}{c}\mathcal{D}_{212}\\ \tilde{\mathcal{E}}(g) = \tilde{c}\end{array}}{\tilde{\mathcal{E}}(n \dot{-} k) = \tilde{c}}\,[\text{i·compute}_1] \quad \begin{array}{c}\mathcal{D}_{22}\\ \tilde{c}^\diamond = c\end{array} \quad \begin{array}{c}\mathcal{D}_{23}\\ \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}\end{array}}{\textit{flatten}\ n\ \tilde{\mathcal{E}}\ (k :: \mathcal{I}) = (c;\mathcal{L})}\,[\text{flatten}_2] \quad \Rightarrow \quad \dfrac{\dfrac{\begin{array}{c}\mathcal{D}_4\\ (n+1) \dot{-} k = g+1\end{array} \quad \dfrac{\begin{array}{c}\mathcal{D}_{212}\\ \tilde{\mathcal{E}}(g) = \tilde{c}\end{array}}{(\tilde{c}';\tilde{\mathcal{E}})(g+1) = \tilde{c}}\,[\text{i·fetch}_2]}{(\tilde{c}';\tilde{\mathcal{E}})(n+1 \dot{-} k) = \tilde{c}}\,[\text{i·compute}_1] \quad \begin{array}{c}\mathcal{D}_{22}\\ \tilde{c}^\diamond = c\end{array} \quad \begin{array}{c}\mathcal{D}_3\\ \textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\ \mathcal{I} = \mathcal{L}\end{array}}{\textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\ (k :: \mathcal{I}) = (c;\mathcal{L})}\,[\text{flatten}_2] \quad ^{[\text{weaken·flatten}]}}\ [\&2]$$

%mode $+\tilde{c}' \cdot +\mathcal{D}_1 \quad \Rightarrow \quad -\mathcal{D}_2 \quad ^{[\text{weaken·flatten}]}$
%worlds $() \quad \tilde{c}' \cdot \mathcal{D}_1 \quad \Rightarrow \quad \mathcal{D}_2 \quad ^{[\text{weaken·flatten}]}$
%total $(\mathcal{D}_1) \quad \tilde{c}' \cdot \mathcal{D}_1 \quad \Rightarrow \quad \mathcal{D}_2 \quad ^{[\text{weaken·flatten}]}$

$\square$

%lemma $\forall \tilde{c}' \cdot \textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ \mathcal{I}_\mu = \mathcal{L}_\mu \quad \Rightarrow \quad \textit{map}\ \left(\textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\right)\ \mathcal{I}_\mu = \mathcal{L}_\mu \quad ^{[\text{weaken·map}]}$

Proof.

$$\dfrac{}{\tilde{c}' : \textit{clos} \cdot \quad \textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ [] = ()\ ^{[\text{map}_1]} \quad \Rightarrow \quad \textit{map}\ \left(\textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\right)\ [] = ()\ ^{[\text{map}_1]} \quad ^{[\text{weaken·map}]}}\ [\&1]$$

$$\dfrac{\begin{array}{c} \tilde{c}' : \textit{clos} \cdot \quad \begin{array}{c}\mathcal{D}_{12}\\ \textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ \mathcal{I}_\mu = \mathcal{L}_\mu\end{array} \quad \Rightarrow \quad \begin{array}{c}\mathcal{D}_2\\ \textit{map}\ \left(\textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\right)\ \mathcal{I}_\mu = \mathcal{L}_\mu\end{array} \quad ^{[\text{weaken·map}]} \\ \tilde{c}' : \textit{clos} \cdot \quad \begin{array}{c}\mathcal{D}_{11}\\ \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}\end{array} \quad \Rightarrow \quad \begin{array}{c}\mathcal{D}_3\\ \textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\ \mathcal{I} = \mathcal{L}\end{array} \quad ^{[\text{weaken·flatten}]} \end{array}}{\tilde{c}' : \textit{clos} \cdot \quad \dfrac{\begin{array}{c}\mathcal{D}_{11}\\ \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}\end{array} \quad \begin{array}{c}\mathcal{D}_{12}\\ \textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ \mathcal{I}_\mu = \mathcal{L}_\mu\end{array}}{\textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ (\mathcal{I} :: \mathcal{I}_\mu) = (\mathcal{L} : \mathcal{L}_\mu)}\,[\text{map}_2] \quad \Rightarrow \quad \dfrac{\begin{array}{c}\mathcal{D}_3\\ \textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\ \mathcal{I} = \mathcal{L}\end{array} \quad \begin{array}{c}\mathcal{D}_2\\ \textit{map}\ \left(\textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\right)\ \mathcal{I}_\mu = \mathcal{L}_\mu\end{array}}{\textit{map}\ \left(\textit{flatten}\ (n+1)\ (\tilde{c}';\tilde{\mathcal{E}})\right)\ (\mathcal{I} :: \mathcal{I}_\mu) = (\mathcal{L} : \mathcal{L}_\mu)}\,[\text{map}_2] \quad ^{[\text{weaken·map}]}}\ [\&2]$$

%mode $+\tilde{c}' \cdot +\mathcal{D}_1 \quad \Rightarrow \quad -\mathcal{D}_2 \quad ^{[\text{weaken·map}]}$
%worlds $() \quad \tilde{c}' \cdot \mathcal{D}_1 \quad \Rightarrow \quad \mathcal{D}_2 \quad ^{[\text{weaken·map}]}$
%total $(\mathcal{D}_1) \quad \tilde{c}' \cdot \mathcal{D}_1 \quad \Rightarrow \quad \mathcal{D}_2 \quad ^{[\text{weaken·map}]}$

$\square$

%lemma $\textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ \mathcal{I}_\mu = \mathcal{L}_\mu \quad \wedge \quad \mathcal{L}_\mu(\alpha) = \mathcal{L}' \quad \Rightarrow \quad \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I}' = \mathcal{L}' \quad \wedge \quad \mathcal{I}_\mu(\alpha) = \mathcal{I}' \quad \textit{for some}\ \mathcal{I}' \quad ^{[\text{map·complete}]}$

Proof.

$$\dfrac{}{\dfrac{\begin{array}{c}\mathcal{D}_{11}\\ \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}\end{array} \quad \begin{array}{c}\vdots\\ \textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ \mathcal{I}_\mu = \mathcal{L}_\mu\end{array}}{\textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ (\mathcal{I} :: \mathcal{I}_\mu) = (\mathcal{L} : \mathcal{L}_\mu)}\,[\text{map}_2] \quad \wedge \quad (\mathcal{L} : \mathcal{L}_\mu)(0) = \mathcal{L}\ ^{[\text{l·fetch}_1]} \quad \Rightarrow \quad \begin{array}{c}\mathcal{D}_{11}\\ \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}\end{array} \quad \wedge \quad (\mathcal{I} :: \mathcal{I}_\mu)(0) = \mathcal{I}\ ^{\left[\text{fetch}_1^{\mathcal{I}_\mu}\right]} \quad ^{[\text{map·complete}]}}\ [\&1]$$

$$\dfrac{\begin{array}{c}\mathcal{D}_{12}\\ \textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ \mathcal{I}_\mu = \mathcal{L}_\mu\end{array} \quad \wedge \quad \begin{array}{c}\mathcal{D}_4\\ \mathcal{L}_\mu(\alpha) = \mathcal{L}'\end{array} \quad \Rightarrow \quad \begin{array}{c}\mathcal{D}_3\\ \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I}' = \mathcal{L}'\end{array} \quad \wedge \quad \begin{array}{c}\mathcal{D}_2\\ \mathcal{I}_\mu(\alpha) = \mathcal{I}'\end{array} \quad ^{[\text{map·complete}]}}{\dfrac{\begin{array}{c}\vdots\\ \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}\end{array} \quad \begin{array}{c}\mathcal{D}_{12}\\ \textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ \mathcal{I}_\mu = \mathcal{L}_\mu\end{array}}{\textit{map}\ \left(\textit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ (\mathcal{I} :: \mathcal{I}_\mu) = (\mathcal{L} : \mathcal{L}_\mu)}\,[\text{map}_2] \quad \wedge \quad \dfrac{\begin{array}{c}\mathcal{D}_4\\ \mathcal{L}_\mu(\alpha) = \mathcal{L}'\end{array}}{(\mathcal{L} : \mathcal{L}_\mu)(\alpha+1) = \mathcal{L}'}\,[\text{l·fetch}_2] \quad \Rightarrow \quad \begin{array}{c}\mathcal{D}_3\\ \textit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I}' = \mathcal{L}'\end{array} \quad \wedge \quad \dfrac{\begin{array}{c}\mathcal{D}_2\\ \mathcal{I}_\mu(\alpha) = \mathcal{I}'\end{array}}{(\mathcal{I} :: \mathcal{I}_\mu)(\alpha+1) = \mathcal{I}'}\,\left[\text{fetch}_2^{\mathcal{I}_\mu}\right] \quad ^{[\text{map·complete}]}}\ [\&2]$$

%mode $+\mathcal{D}_1 \quad \wedge \quad +\mathcal{D}_2 \quad \Rightarrow \quad -\mathcal{D}_3 \quad \wedge \quad -\mathcal{D}_4 \quad ^{[\text{map·complete}]}$
%worlds $() \quad \mathcal{D}_1 \quad \wedge \quad \mathcal{D}_2 \quad \Rightarrow \quad \mathcal{D}_3 \quad \wedge \quad \mathcal{D}_4 \quad ^{[\text{map·complete}]}$

%total $(\mathcal{D}_2)\;\; \mathcal{D}_1 \;\wedge\; \mathcal{D}_2 \;\Rightarrow\; \mathcal{D}_3 \;\wedge\; \mathcal{D}_4$ [map·complete] □

%lemma $\textit{flatten}\; n\; \tilde{\mathcal{E}}\; \mathcal{I} = \mathcal{L} \;\wedge\; \mathcal{L}(l) = c \;\Rightarrow\; \tilde{\mathcal{E}}(n \mathbin{\dot{-}} k) = \tilde{c} \;\wedge\; \tilde{c}^{\diamond} = c \;\wedge\; \mathcal{I}(l) = k \;\; \textit{for some } k, \tilde{c}$ [fetch·complete]

Proof.

[&1]

$$\dfrac{\overset{\mathcal{D}_{31}}{\tilde{\mathcal{E}}(n \mathbin{\dot{-}} k) = \tilde{c}} \quad \overset{\mathcal{D}_{32}}{\tilde{c}^{\diamond} = c} \quad \overset{\vdots}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; \mathcal{I} = \mathcal{L}}}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; (k :: \mathcal{I}) = (c; \mathcal{L})}\,[\text{flatten}_2] \;\wedge\; (c;\mathcal{L})(0) = c\,[\text{fetch}_1] \;\Rightarrow\; \overset{\mathcal{D}_{31}}{\tilde{\mathcal{E}}(n \mathbin{\dot{-}} k) = \tilde{c}} \;\wedge\; \overset{\mathcal{D}_{32}}{\tilde{c}^{\diamond} = c} \;\wedge\; (k :: \mathcal{I})(0) = k\,[\text{fetch}^{\mathcal{I}}_1] \quad [\text{fetch·complete}]$$

$$\overset{\mathcal{D}_{33}}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; \mathcal{I} = \mathcal{L}} \;\wedge\; \overset{\mathcal{D}_5}{\mathcal{L}(l) = c} \;\Rightarrow\; \overset{\mathcal{D}_2}{\tilde{\mathcal{E}}(n \mathbin{\dot{-}} k) = \tilde{c}} \;\wedge\; \overset{\mathcal{D}_4}{\tilde{c}^{\diamond} = c} \;\wedge\; \overset{\mathcal{D}_{11}}{\mathcal{I}(l) = k} \quad [\text{fetch·complete}]$$

[&2]

$$\dfrac{\overset{\mathcal{D}_{31}}{\tilde{\mathcal{E}}(n \mathbin{\dot{-}} k') = \tilde{c}'} \quad \overset{\mathcal{D}_{32}}{\tilde{c}'^{\diamond} = c'} \quad \overset{\mathcal{D}_{33}}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; \mathcal{I} = \mathcal{L}}}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; (k' :: \mathcal{I}) = (c'; \mathcal{L})}\,[\text{flatten}_2] \;\wedge\; \dfrac{\overset{\mathcal{D}_5}{\mathcal{L}(l) = c}}{(c';\mathcal{L})(l+1) = c}\,[\text{fetch}_2] \;\Rightarrow\; \overset{\mathcal{D}_2}{\tilde{\mathcal{E}}(n \mathbin{\dot{-}} k) = \tilde{c}} \;\wedge\; \overset{\mathcal{D}_4}{\tilde{c}^{\diamond} = c} \;\wedge\; \dfrac{\overset{\mathcal{D}_{11}}{\mathcal{I}(l) = k}}{(k' :: \mathcal{I})(l+1) = k}\,[\text{fetch}^{\mathcal{I}}_2] \quad [\text{fetch·complete}]$$

%mode $+\mathcal{D}_1 \;\wedge\; +\mathcal{D}_2 \;\Rightarrow\; -\mathcal{D}_3 \;\wedge\; -\mathcal{D}_4 \;\wedge\; -\mathcal{D}_5$ [fetch·complete]
%worlds $()\;\; \mathcal{D}_1 \;\wedge\; \mathcal{D}_2 \;\Rightarrow\; \mathcal{D}_3 \;\wedge\; \mathcal{D}_4 \;\wedge\; \mathcal{D}_5$ [fetch·complete]
%total $(\mathcal{D}_1)\;\; \mathcal{D}_1 \;\wedge\; \mathcal{D}_2 \;\Rightarrow\; \mathcal{D}_3 \;\wedge\; \mathcal{D}_4 \;\wedge\; \mathcal{D}_5$ [fetch·complete] □

%lemma $\tilde{\mathcal{E}}_{\mu}{}^{\diamond} = \mathcal{E}_{\mu} \;\wedge\; \mathcal{E}_{\mu}(\alpha) = \mathcal{S} \;\Rightarrow\; \tilde{\mathcal{S}}^{\diamond} = \mathcal{S} \;\wedge\; \tilde{\mathcal{E}}_{\mu}(\alpha) = \tilde{\mathcal{S}} \;\; \textit{for some } \tilde{\mathcal{S}}$ [fetch$^{\mu}$·complete]

Proof.

[&1]

$$\dfrac{\overset{\mathcal{D}_{11}}{\tilde{\mathcal{S}}^{\diamond} = \mathcal{S}} \quad \overset{\vdots}{\tilde{\mathcal{E}}_{\mu}{}^{\diamond} = \mathcal{E}_{\mu}}}{(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_{\mu})^{\diamond} = (\mathcal{S};\mathcal{E}_{\mu})}\,[\text{k·env}^{\diamond}_2] \;\wedge\; (\mathcal{S};\mathcal{E}_{\mu})(0) = \mathcal{S}\,[\text{fetch}^{\mu}_1] \;\Rightarrow\; \overset{\mathcal{D}_{11}}{\tilde{\mathcal{S}}^{\diamond} = \mathcal{S}} \;\wedge\; (\tilde{\mathcal{S}};\tilde{\mathcal{E}}_{\mu})(0) = \tilde{\mathcal{S}}\,[\text{i·fetch}^{\mu}_1] \quad [\text{fetch}^{\mu}\text{·complete}]$$

$$\overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}_{\mu}{}^{\diamond} = \mathcal{E}_{\mu}} \;\wedge\; \overset{\mathcal{D}_4}{\mathcal{E}_{\mu}(\alpha) = \mathcal{S}} \;\Rightarrow\; \overset{\mathcal{D}_3}{\tilde{\mathcal{S}}^{\diamond} = \mathcal{S}} \;\wedge\; \overset{\mathcal{D}_2}{\tilde{\mathcal{E}}_{\mu}(\alpha) = \tilde{\mathcal{S}}} \quad [\text{fetch}^{\mu}\text{·complete}]$$

[&2]

$$\dfrac{\overset{\vdots}{\tilde{\mathcal{S}}'^{\diamond} = \mathcal{S}'} \quad \overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}_{\mu}{}^{\diamond} = \mathcal{E}_{\mu}}}{(\tilde{\mathcal{S}}';\tilde{\mathcal{E}}_{\mu})^{\diamond} = (\mathcal{S}';\mathcal{E}_{\mu})}\,[\text{k·env}^{\diamond}_2] \;\wedge\; \dfrac{\overset{\mathcal{D}_4}{\mathcal{E}_{\mu}(\alpha) = \mathcal{S}}}{(\mathcal{S}';\mathcal{E}_{\mu})(\alpha+1) = \mathcal{S}}\,[\text{fetch}^{\mu}_2] \;\Rightarrow\; \overset{\mathcal{D}_3}{\tilde{\mathcal{S}}^{\diamond} = \mathcal{S}} \;\wedge\; \dfrac{\overset{\mathcal{D}_2}{\tilde{\mathcal{E}}_{\mu}(\alpha) = \tilde{\mathcal{S}}}}{(\tilde{\mathcal{S}}';\tilde{\mathcal{E}}_{\mu})(\alpha+1) = \tilde{\mathcal{S}}}\,[\text{i·fetch}^{\mu}_2] \quad [\text{fetch}^{\mu}\text{·complete}]$$

%mode $+\mathcal{D}_1 \;\wedge\; +\mathcal{D}_2 \;\Rightarrow\; -\mathcal{D}_3 \;\wedge\; -\mathcal{D}_4$ [fetch$^{\mu}$·complete]
%worlds $()\;\; \mathcal{D}_1 \;\wedge\; \mathcal{D}_2 \;\Rightarrow\; \mathcal{D}_3 \;\wedge\; \mathcal{D}_4$ [fetch$^{\mu}$·complete]
%total $(\mathcal{D}_2)\;\; \mathcal{D}_1 \;\wedge\; \mathcal{D}_2 \;\Rightarrow\; \mathcal{D}_3 \;\wedge\; \mathcal{D}_4$ [fetch$^{\mu}$·complete] □

%theorem $\sigma_1 \leadsto \sigma_2 \;\wedge\; \tilde{\sigma}_1{}^{\diamond} = \sigma_1 \;\Rightarrow\; \tilde{\sigma}_1 \leadsto \tilde{\sigma}_2 \;\wedge\; \tilde{\sigma}_2{}^{\diamond} = \sigma_2 \;\; \textit{for some } \tilde{\sigma}_2$ [completeness]

Proof.

$$\overset{\mathcal{D}_{211}}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; \mathcal{I} = \mathcal{L}} \;\wedge\; \overset{\mathcal{D}_4}{\mathcal{L}(l) = (t', \mathcal{L}', \mathcal{L}'_{\mu}, \mathcal{E}'_{\mu})} \;\Rightarrow\; \dfrac{\overset{\mathcal{D}_{112}}{n \mathbin{\dot{-}} k = g} \quad \overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}(g) = (t, n', \mathcal{I}', \mathcal{I}'_{\mu}, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_{\mu})}}{\tilde{\mathcal{E}}(n \mathbin{\dot{-}} k) = (t, n', \mathcal{I}', \mathcal{I}'_{\mu}, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_{\mu})}\,[\text{i·compute}_1] \;\wedge\; \overset{\mathcal{D}_{31}}{(t, n', \mathcal{I}', \mathcal{I}'_{\mu}, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_{\mu})^{\diamond} = (t', \mathcal{L}', \mathcal{L}'_{\mu}, \mathcal{E}'_{\mu})} \;\wedge\; \overset{\mathcal{D}_{111}}{\mathcal{I}(l) = k} \quad [\text{fetch·complete}]$$

[&1]

$$\dfrac{\overset{\mathcal{D}_4}{\mathcal{L}(l) = (t', \mathcal{L}', \mathcal{L}'_{\mu}, \mathcal{E}'_{\mu})}}{\langle l, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu}, \mathcal{S}\rangle \leadsto \langle t', \mathcal{L}', \mathcal{L}'_{\mu}, \mathcal{E}'_{\mu}, \mathcal{S}\rangle}\,[\text{k·var}] \;\wedge\; \dfrac{\dfrac{\overset{\mathcal{D}_{211}}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; \mathcal{I} = \mathcal{L}} \quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\; n\; \tilde{\mathcal{E}})\; \mathcal{I}_{\mu} = \mathcal{L}_{\mu}} \quad \overset{\vdots}{\tilde{\mathcal{E}}_{\mu}{}^{\diamond} = \mathcal{E}_{\mu}}}{(l, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu})^{\diamond} = (l, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu})}\,[\text{clos}^{\diamond}] \quad \overset{\mathcal{D}_{22}}{\tilde{\mathcal{S}}^{\diamond} = \mathcal{S}}}{\langle l, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}, \tilde{\mathcal{S}}\rangle^{\diamond} = \langle l, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu}, \mathcal{S}\rangle}\,[\text{state}^{\diamond}]$$

$$\Rightarrow\; \dfrac{\dfrac{\overset{\mathcal{D}_{111}}{\mathcal{I}(l) = k} \quad \overset{\mathcal{D}_{112}}{n \mathbin{\dot{-}} k = g}}{n \mathbin{\dot{-}} \mathcal{I}(l) = g}\,[\text{compute}_1] \quad \overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}(g) = (t, n', \mathcal{I}', \mathcal{I}'_{\mu}, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_{\mu})}}{\langle l, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}, \tilde{\mathcal{S}}\rangle \leadsto \langle t, n', \mathcal{I}', \mathcal{I}'_{\mu}, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_{\mu}, \tilde{\mathcal{S}}\rangle}\,[\text{i·var}] \;\wedge\; \dfrac{\overset{\mathcal{D}_{31}}{(t, n', \mathcal{I}', \mathcal{I}'_{\mu}, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_{\mu})^{\diamond} = (t', \mathcal{L}', \mathcal{L}'_{\mu}, \mathcal{E}'_{\mu})} \quad \overset{\mathcal{D}_{22}}{\tilde{\mathcal{S}}^{\diamond} = \mathcal{S}}}{\langle t, n', \mathcal{I}', \mathcal{I}'_{\mu}, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_{\mu}, \tilde{\mathcal{S}}\rangle^{\diamond} = \langle t', \mathcal{L}', \mathcal{L}'_{\mu}, \mathcal{E}'_{\mu}, \mathcal{S}\rangle}\,[\text{state}^{\diamond}] \quad [\text{completeness}]$$

[&2]

$$\langle (t\,u), \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu}, \mathcal{S}\rangle \leadsto \langle t, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu}, (u, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu}) :: \mathcal{S}\rangle\,[\text{k·app}] \;\wedge\; \dfrac{\dfrac{\overset{\mathcal{D}_{211}}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; \mathcal{I} = \mathcal{L}} \quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\; n\; \tilde{\mathcal{E}})\; \mathcal{I}_{\mu} = \mathcal{L}_{\mu}} \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_{\mu}{}^{\diamond} = \mathcal{E}_{\mu}}}{((t\,u), n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu})^{\diamond} = ((t\,u), \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu})}\,[\text{clos}^{\diamond}] \quad \overset{\mathcal{D}_{23}}{\tilde{\mathcal{S}}^{\diamond} = \mathcal{S}}}{\langle (t\,u), n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}, \tilde{\mathcal{S}}\rangle^{\diamond} = \langle (t\,u), \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu}, \mathcal{S}\rangle}\,[\text{state}^{\diamond}]$$

$$\Rightarrow\; \langle (t\,u), n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}, \tilde{\mathcal{S}}\rangle \leadsto \langle t, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}, (u, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}) :: \tilde{\mathcal{S}}\rangle\,[\text{i·app}] \;\wedge\; \dfrac{\dfrac{\overset{\mathcal{D}_{211}}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; \mathcal{I} = \mathcal{L}} \quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\; n\; \tilde{\mathcal{E}})\; \mathcal{I}_{\mu} = \mathcal{L}_{\mu}} \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_{\mu}{}^{\diamond} = \mathcal{E}_{\mu}}}{(t, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu})^{\diamond} = (t, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu})}\,[\text{clos}^{\diamond}] \quad \dfrac{\dfrac{\overset{\mathcal{D}_{211}}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; \mathcal{I} = \mathcal{L}} \quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\; n\; \tilde{\mathcal{E}})\; \mathcal{I}_{\mu} = \mathcal{L}_{\mu}} \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_{\mu}{}^{\diamond} = \mathcal{E}_{\mu}}}{(u, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu})^{\diamond} = (u, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu})}\,[\text{clos}^{\diamond}] \quad \overset{\mathcal{D}_{23}}{\tilde{\mathcal{S}}^{\diamond} = \mathcal{S}}}{((u, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}) :: \tilde{\mathcal{S}})^{\diamond} = (u, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu}) :: \mathcal{S}}\,[\text{stack}^{\diamond}_2]}{\langle t, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}, (u, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}) :: \tilde{\mathcal{S}}\rangle^{\diamond} = \langle t, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu}, (u, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu}) :: \mathcal{S}\rangle}\,[\text{state}^{\diamond}] \quad [\text{completeness}]$$

$$n+1 : \textit{index} \cdot\; \overset{\mathcal{D}_1}{(n+1) \mathbin{\dot{-}} (n+1) = 0} \quad [\text{minus·equals}]$$

$$\tilde{c} : \textit{clos} \cdot\; \overset{\mathcal{D}_{211}}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; \mathcal{I} = \mathcal{L}} \;\Rightarrow\; \overset{\mathcal{D}_2}{\textit{flatten}\; (n+1)\; (\tilde{c};\tilde{\mathcal{E}})\; \mathcal{I} = \mathcal{L}} \quad [\text{weaken·flatten}]$$

$$\tilde{c} : \textit{clos} \cdot\; \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\; n\; \tilde{\mathcal{E}})\; \mathcal{I}_{\mu} = \mathcal{L}_{\mu}} \;\Rightarrow\; \overset{\mathcal{D}_3}{\textit{map}\;(\textit{flatten}\; (n+1)\; (\tilde{c};\tilde{\mathcal{E}}))\; \mathcal{I}_{\mu} = \mathcal{L}_{\mu}} \quad [\text{weaken·map}]$$

[&3]

$$\langle \lambda t, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu}, c :: \mathcal{S}\rangle \leadsto \langle t, (c;\mathcal{L}), \mathcal{L}_{\mu}, \mathcal{E}_{\mu}, \mathcal{S}\rangle\,[\text{k·abs}] \;\wedge\; \dfrac{\dfrac{\overset{\mathcal{D}_{211}}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; \mathcal{I} = \mathcal{L}} \quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\; n\; \tilde{\mathcal{E}})\; \mathcal{I}_{\mu} = \mathcal{L}_{\mu}} \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_{\mu}{}^{\diamond} = \mathcal{E}_{\mu}}}{(\lambda t, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu})^{\diamond} = (\lambda t, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu})}\,[\text{clos}^{\diamond}] \quad \dfrac{\overset{\mathcal{D}_{221}}{\tilde{c}^{\diamond} = c} \quad \overset{\mathcal{D}_{222}}{\tilde{\mathcal{S}}^{\diamond} = \mathcal{S}}}{(\tilde{c} :: \tilde{\mathcal{S}})^{\diamond} = c :: \mathcal{S}}\,[\text{stack}^{\diamond}_2]}{\langle \lambda t, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}, \tilde{c} :: \tilde{\mathcal{S}}\rangle^{\diamond} = \langle \lambda t, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu}, c :: \mathcal{S}\rangle}\,[\text{state}^{\diamond}]$$

$$\Rightarrow\; \langle \lambda t, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}, \tilde{c} :: \tilde{\mathcal{S}}\rangle \leadsto \langle t, n+1, (n+1 :: \mathcal{I}), \mathcal{I}_{\mu}, (\tilde{c};\tilde{\mathcal{E}}), \tilde{\mathcal{E}}_{\mu}, \tilde{\mathcal{S}}\rangle\,[\text{i·abs}] \;\wedge\; \dfrac{\dfrac{\dfrac{\dfrac{\overset{\mathcal{D}_1}{(n+1) \mathbin{\dot{-}} (n+1) = 0} \quad (\tilde{c};\tilde{\mathcal{E}})(0) = \tilde{c}\,[\text{i·fetch}_1]}{(\tilde{c};\tilde{\mathcal{E}})((n+1) \mathbin{\dot{-}} (n+1)) = \tilde{c}}\,[\text{i·compute}_1] \quad \overset{\mathcal{D}_{221}}{\tilde{c}^{\diamond} = c} \quad \overset{\mathcal{D}_2}{\textit{flatten}\; (n+1)\; (\tilde{c};\tilde{\mathcal{E}})\; \mathcal{I} = \mathcal{L}}}{\textit{flatten}\; (n+1)\; (\tilde{c};\tilde{\mathcal{E}})\; (n+1 :: \mathcal{I}) = (c;\mathcal{L})}\,[\text{flatten}_2] \quad \overset{\mathcal{D}_3}{\textit{map}\;(\textit{flatten}\; (n+1)\; (\tilde{c};\tilde{\mathcal{E}}))\; \mathcal{I}_{\mu} = \mathcal{L}_{\mu}} \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_{\mu}{}^{\diamond} = \mathcal{E}_{\mu}}}{(t, n+1, (n+1 :: \mathcal{I}), \mathcal{I}_{\mu}, (\tilde{c};\tilde{\mathcal{E}}), \tilde{\mathcal{E}}_{\mu})^{\diamond} = (t, (c;\mathcal{L}), \mathcal{L}_{\mu}, \mathcal{E}_{\mu})}\,[\text{clos}^{\diamond}] \quad \overset{\mathcal{D}_{222}}{\tilde{\mathcal{S}}^{\diamond} = \mathcal{S}}}{\langle t, n+1, (n+1 :: \mathcal{I}), \mathcal{I}_{\mu}, (\tilde{c};\tilde{\mathcal{E}}), \tilde{\mathcal{E}}_{\mu}, \tilde{\mathcal{S}}\rangle^{\diamond} = \langle t, (c;\mathcal{L}), \mathcal{L}_{\mu}, \mathcal{E}_{\mu}, \mathcal{S}\rangle}\,[\text{state}^{\diamond}] \quad [\text{completeness}]$$

[&4]

$$\langle \textbf{get-context}\; t, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu}, \mathcal{S}\rangle \leadsto \langle t, \mathcal{L}, (\mathcal{L} \!:\! \mathcal{L}_{\mu}), (\mathcal{S};\mathcal{E}_{\mu}), \mathcal{S}\rangle\,[\text{k·catch}] \;\wedge\; \dfrac{\dfrac{\overset{\mathcal{D}_{211}}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; \mathcal{I} = \mathcal{L}} \quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\; n\; \tilde{\mathcal{E}})\; \mathcal{I}_{\mu} = \mathcal{L}_{\mu}} \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_{\mu}{}^{\diamond} = \mathcal{E}_{\mu}}}{(\textbf{get-context}\; t, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu})^{\diamond} = (\textbf{get-context}\; t, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu})}\,[\text{clos}^{\diamond}] \quad \overset{\mathcal{D}_{22}}{\tilde{\mathcal{S}}^{\diamond} = \mathcal{S}}}{\langle \textbf{get-context}\; t, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}, \tilde{\mathcal{S}}\rangle^{\diamond} = \langle \textbf{get-context}\; t, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu}, \mathcal{S}\rangle}\,[\text{state}^{\diamond}]$$

$$\Rightarrow\; \langle \textbf{get-context}\; t, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}, \tilde{\mathcal{S}}\rangle \leadsto \langle t, n, \mathcal{I}, (\mathcal{I} :: \mathcal{I}_{\mu}), \tilde{\mathcal{E}}, (\tilde{\mathcal{S}};\tilde{\mathcal{E}}_{\mu}), \tilde{\mathcal{S}}\rangle\,[\text{i·catch}] \;\wedge\; \dfrac{\dfrac{\overset{\mathcal{D}_{211}}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; \mathcal{I} = \mathcal{L}} \quad \dfrac{\overset{\mathcal{D}_{211}}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; \mathcal{I} = \mathcal{L}} \quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\; n\; \tilde{\mathcal{E}})\; \mathcal{I}_{\mu} = \mathcal{L}_{\mu}}}{\textit{map}\;(\textit{flatten}\; n\; \tilde{\mathcal{E}})\; (\mathcal{I} :: \mathcal{I}_{\mu}) = (\mathcal{L} \!:\! \mathcal{L}_{\mu})}\,[\text{map}_2] \quad \dfrac{\overset{\mathcal{D}_{22}}{\tilde{\mathcal{S}}^{\diamond} = \mathcal{S}} \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_{\mu}{}^{\diamond} = \mathcal{E}_{\mu}}}{(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_{\mu})^{\diamond} = (\mathcal{S};\mathcal{E}_{\mu})}\,[\text{k·env}^{\diamond}_2]}{(t, n, \mathcal{I}, (\mathcal{I} :: \mathcal{I}_{\mu}), \tilde{\mathcal{E}}, (\tilde{\mathcal{S}};\tilde{\mathcal{E}}_{\mu}))^{\diamond} = (t, \mathcal{L}, (\mathcal{L} \!:\! \mathcal{L}_{\mu}), (\mathcal{S};\mathcal{E}_{\mu}))}\,[\text{clos}^{\diamond}] \quad \overset{\mathcal{D}_{22}}{\tilde{\mathcal{S}}^{\diamond} = \mathcal{S}}}{\langle t, n, \mathcal{I}, (\mathcal{I} :: \mathcal{I}_{\mu}), \tilde{\mathcal{E}}, (\tilde{\mathcal{S}};\tilde{\mathcal{E}}_{\mu}), \tilde{\mathcal{S}}\rangle^{\diamond} = \langle t, \mathcal{L}, (\mathcal{L} \!:\! \mathcal{L}_{\mu}), (\mathcal{S};\mathcal{E}_{\mu}), \mathcal{S}\rangle}\,[\text{state}^{\diamond}] \quad [\text{completeness}]$$

$$\overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_{\mu}{}^{\diamond} = \mathcal{E}_{\mu}} \;\wedge\; \overset{\mathcal{D}_4}{\mathcal{E}_{\mu}(\alpha) = \mathcal{S}'} \;\Rightarrow\; \overset{\mathcal{D}_{32}}{\tilde{\mathcal{S}}'^{\diamond} = \mathcal{S}'} \;\wedge\; \overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}_{\mu}(\alpha) = \tilde{\mathcal{S}}'} \quad [\text{fetch}^{\mu}\text{·complete}]$$

$$\overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\; n\; \tilde{\mathcal{E}})\; \mathcal{I}_{\mu} = \mathcal{L}_{\mu}} \;\wedge\; \overset{\mathcal{D}_6}{\mathcal{L}_{\mu}(\alpha) = \mathcal{L}'} \;\Rightarrow\; \overset{\mathcal{D}_5}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; \mathcal{I}' = \mathcal{L}'} \;\wedge\; \overset{\mathcal{D}_{11}}{\mathcal{I}_{\mu}(\alpha) = \mathcal{I}'} \quad [\text{map·complete}]$$

[&5]

$$\dfrac{\overset{\mathcal{D}_6}{\mathcal{L}_{\mu}(\alpha) = \mathcal{L}'} \quad \overset{\mathcal{D}_4}{\mathcal{E}_{\mu}(\alpha) = \mathcal{S}'}}{\langle \textbf{set-context}\; \alpha\; t, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu}, \mathcal{S}\rangle \leadsto \langle t, \mathcal{L}', \mathcal{L}_{\mu}, \mathcal{E}_{\mu}, \mathcal{S}'\rangle}\,[\text{k·throw}] \;\wedge\; \dfrac{\dfrac{\overset{\vdots}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; \mathcal{I} = \mathcal{L}} \quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\; n\; \tilde{\mathcal{E}})\; \mathcal{I}_{\mu} = \mathcal{L}_{\mu}} \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_{\mu}{}^{\diamond} = \mathcal{E}_{\mu}}}{(\textbf{set-context}\; \alpha\; t, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu})^{\diamond} = (\textbf{set-context}\; \alpha\; t, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu})}\,[\text{clos}^{\diamond}] \quad \overset{\vdots}{\tilde{\mathcal{S}}^{\diamond} = \mathcal{S}}}{\langle \textbf{set-context}\; \alpha\; t, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}, \tilde{\mathcal{S}}\rangle^{\diamond} = \langle \textbf{set-context}\; \alpha\; t, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu}, \mathcal{S}\rangle}\,[\text{state}^{\diamond}]$$

$$\Rightarrow\; \dfrac{\overset{\mathcal{D}_{11}}{\mathcal{I}_{\mu}(\alpha) = \mathcal{I}'} \quad \overset{\mathcal{D}_{12}}{\tilde{\mathcal{E}}_{\mu}(\alpha) = \tilde{\mathcal{S}}'}}{\langle \textbf{set-context}\; \alpha\; t, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}, \tilde{\mathcal{S}}\rangle \leadsto \langle t, n, \mathcal{I}', \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}, \tilde{\mathcal{S}}'\rangle}\,[\text{i·throw}] \;\wedge\; \dfrac{\dfrac{\overset{\mathcal{D}_5}{\textit{flatten}\; n\; \tilde{\mathcal{E}}\; \mathcal{I}' = \mathcal{L}'} \quad \overset{\mathcal{D}_{212}}{\textit{map}\;(\textit{flatten}\; n\; \tilde{\mathcal{E}})\; \mathcal{I}_{\mu} = \mathcal{L}_{\mu}} \quad \overset{\mathcal{D}_{213}}{\tilde{\mathcal{E}}_{\mu}{}^{\diamond} = \mathcal{E}_{\mu}}}{(t, n, \mathcal{I}', \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu})^{\diamond} = (t, \mathcal{L}', \mathcal{L}_{\mu}, \mathcal{E}_{\mu})}\,[\text{clos}^{\diamond}] \quad \overset{\mathcal{D}_{32}}{\tilde{\mathcal{S}}'^{\diamond} = \mathcal{S}'}}{\langle t, n, \mathcal{I}', \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}, \tilde{\mathcal{S}}'\rangle^{\diamond} = \langle t, \mathcal{L}', \mathcal{L}_{\mu}, \mathcal{E}_{\mu}, \mathcal{S}'\rangle}\,[\text{state}^{\diamond}] \quad [\text{completeness}]$$

%mode $+\mathcal{D}_1 \;\wedge\; +\mathcal{D}_2 \;\Rightarrow\; -\mathcal{D}_3 \;\wedge\; -\mathcal{D}_4$ [completeness]
%worlds $()\;\; \mathcal{D}_1 \;\wedge\; \mathcal{D}_2 \;\Rightarrow\; \mathcal{D}_3 \;\wedge\; \mathcal{D}_4$ [completeness]
%total $(\mathcal{D}_1)\;\; \mathcal{D}_1 \;\wedge\; \mathcal{D}_2 \;\Rightarrow\; \mathcal{D}_3 \;\wedge\; \mathcal{D}_4$ [completeness] □