1 System T

1.1 Syntax

idents : **type**.
number : **type**.
expression : **type**.
expressions : **type**.
block : **type**.
command : **type**.
sequence : **type**.
prop : **type**.
props : **type**.
env : **type**.

1.1.1 Identifiers x

: *idents*.
$\sqcup\,,\,\sqcup$: *idents* $\to$ *ident* $\to$ *idents*.

1.1.2 Environment Γ

$$
\begin{array}{lll}
\Gamma & ::= & \\
 & | & \{\} \\
 & | & \Gamma, x{:}\,\tau
\end{array}
$$

1.1.3 Expression e

$$
\begin{array}{lll}
e & ::= & \\
 & | & x \\
 & | & \star \\
 & | & q \\
 & | & \mathbf{proc}\,[\gamma]\,\mathbf{out}\,[\omega]\,\{s\}
\end{array}
$$

1.1.4 Expressions $\vec{e}$

$$
\begin{array}{lll}
\vec{e} & ::= & \\
 & | & \\
 & | & \vec{e}\,,e
\end{array}
$$

1.1.5 Block b

$$
\begin{array}{lll}
b & ::= & \\
 & | & \{s\}_\omega
\end{array}
$$

1.1.6 Command c

$$
\begin{array}{lll}
c & ::= & \\
 & | & b \\
 & | & \mathbf{for}\,x := 0\,\mathbf{until}\,e\,b \\
 & | & x := e \\
 & | & \mathbf{inc}\,(x) \\
 & | & \mathbf{dec}\,(x) \\
 & | & e(\vec{e}\,;\vec{x}\,)
\end{array}
$$

1.1.7 Sequence s

$$\begin{array}{lll} s & ::= & \\ & | & \varepsilon \\ & | & c;\, s \\ & | & \mathbf{cst}\; x := e;\, s \\ & | & \mathbf{var}\; x := e;\, s \end{array}$$

%abbrev $\mathbf{var}\; x;\, s \quad = \quad \mathbf{var}\; x := \star;\, s.$

1.1.8 Proposition τ

$$\begin{array}{lll} \tau & ::= & \\ & | & \top \\ & | & \mathbf{nat} \\ & | & \mathbf{proc}([\vec{\tau}_1]\, \mathbf{out}\, [\vec{\tau}_2]) \end{array}$$

1.1.9 Propositions $\vec{\tau}$

$$\begin{array}{lll} \vec{\tau} & ::= & \\ & | & \\ & | & \vec{\tau}\,, \tau \end{array}$$

1.2 Typing

Type equality

$\tau_1 = \tau_2 \quad : \quad \mathbf{type}.$

$$\frac{}{\tau = \tau}\,[\text{prop\_eq\_refl}]$$

Lookup

$\sqcup : \sqcup \in \sqcup \quad : \quad \mathit{ident} \;\rightarrow\; \mathit{prop} \;\rightarrow\; \mathit{env} \;\rightarrow\; \mathbf{type}.$

$$\frac{}{x : \tau \in \Sigma, x : \tau}\,[\text{lookup\_i}]$$

$$\frac{x \neq y \quad x : \tau \in \Sigma}{x : \tau \in \Sigma, y : \tau'}\,[\text{lookup\_ii}]$$

Typing judgments

$\langle \mathsf{fenv\#0} \rangle \vdash \langle \mathsf{term\#0} \rangle : \langle \mathsf{typ\#0} \rangle \quad : \quad \mathbf{type}.$
$\langle \mathsf{fenv\#0} \rangle \vdash (\langle \mathsf{terms\#0} \rangle) : (\langle \mathsf{typs\#0} \rangle) \quad : \quad \mathbf{type}.$
$\langle \mathsf{fenv\#1} \rangle, \vec{x} : \langle \mathsf{typs\#0} \rangle = \langle \mathsf{fenv\#2} \rangle \quad : \quad \mathbf{type}.$
$\langle \mathsf{fenv\#0} \rangle, \langle \vec{x} \rangle : \langle \langle \mathsf{typs\#0} \rangle \rangle \vdash \langle \mathsf{term\#0} \rangle : \langle \mathsf{typ\#0} \rangle \quad : \quad \mathbf{type}.$

Type check

$$\frac{\langle \mathsf{f\_lookup} | x | \tau | \Sigma \rangle}{\Sigma \vdash \langle \mathsf{t\_var} | x \rangle : \tau}\,[\text{tc\_var}]$$

$$\frac{}{\Sigma \vdash \langle \mathsf{t\_zero} \rangle : \langle \mathsf{typ\_nat} \rangle}\ [\text{tc\_zero}]$$

$$\frac{\Sigma \vdash t : \langle \mathsf{typ\_nat} \rangle}{\Sigma \vdash \langle \mathsf{t\_succ} | t \rangle : \langle \mathsf{typ\_nat} \rangle}\ [\text{tc\_succ}]$$

$$\frac{\Sigma \vdash t : \langle \mathsf{typ\_nat} \rangle}{\Sigma \vdash \langle \mathsf{t\_pred} | t \rangle : \langle \mathsf{typ\_nat} \rangle}\ [\text{tc\_pred}]$$

$$\frac{\langle \mathsf{f\_env\_cons} | \Sigma | x | \tau \rangle \vdash t : \tau'}{\Sigma \vdash \langle \mathsf{t\_lam} | x | \tau | t \rangle : \langle \mathsf{typ\_imp} | \tau | \tau' \rangle}\ [\text{tc\_lam}]$$

$$\frac{\Sigma \vdash t_1 : \langle \mathsf{typ\_imp} | \tau | \tau' \rangle \quad \Sigma \vdash t_2 : \tau}{\Sigma \vdash \langle \mathsf{t\_app} | t_1 | t_2 \rangle : \tau'}\ [\text{tc\_app}]$$

$$\frac{\Sigma \vdash t_1 : \langle \mathsf{typ\_nat} \rangle \quad \Sigma \vdash t_2 : \tau \quad \Sigma \vdash t_3 : \langle \mathsf{typ\_imp} | \langle \mathsf{typ\_nat} \rangle | (\langle \mathsf{typ\_imp} | \tau | \tau \rangle) \rangle}{\Sigma \vdash \langle \mathsf{t\_rec} | t_1 | t_2 | t_3 \rangle : \tau}\ [\text{tc\_rec}]$$

$$\frac{\Sigma \vdash (\vec{t}\,) : (\vec{\tau}\,)}{\Sigma \vdash \langle \mathsf{t\_tuple} | \vec{t}\, \rangle : \langle \mathsf{typ\_tuple} | \vec{\tau} \rangle}\ [\text{tc\_tuple}]$$

$$\frac{\Sigma \vdash t_1 : \tau \quad \langle \mathsf{f\_env\_cons} | \Sigma | y | \tau \rangle \vdash t_2 : \tau'}{\Sigma \vdash \langle \mathsf{t\_let} | y | t_1 | t_2 \rangle : \tau'}\ [\text{tc\_let}]$$

$$\frac{\Sigma \vdash t_1 : \langle \mathsf{typ\_tuple} | \vec{\tau} \rangle \quad \Sigma, \langle \vec{x}\, \rangle : \langle \vec{\tau}\, \rangle \vdash t_2 : \tau'}{\Sigma \vdash \langle \mathsf{t\_match} | \vec{x}\, | t_1 | t_2 \rangle : \tau'}\ [\text{tc\_match}]$$

Append

$$\frac{}{\Sigma, () : (\langle \mathsf{typs\_empty} \rangle) = \Sigma}\ [\text{app\_i}]$$

$$\frac{\Sigma, \vec{x} : \vec{\tau} = \Sigma'}{\Sigma, (\vec{x}\,, x) : (\langle \mathsf{typs\_cons} | \vec{\tau}\, | \tau \rangle) = \langle \mathsf{f\_env\_cons} | \Sigma' | x | \tau \rangle}\ [\text{app\_ii}]$$

Type check terms in extended environment

$$\frac{\Sigma, \vec{x} : \vec{\tau} = \Sigma' \quad \Sigma' \vdash t : \tau'}{\Sigma, \langle \vec{x}\, \rangle : \langle \vec{\tau}\, \rangle \vdash t : \tau'}\ [\text{tcte\_product}]$$

Type check terms

$$\frac{}{\Sigma \vdash (\langle \mathsf{ts\_empty} \rangle) : (\langle \mathsf{typs\_empty} \rangle)}\ [\text{tcts\_empty}]$$

$$\frac{\Sigma \vdash t : \tau \quad \Sigma \vdash (\vec{t}\,) : (\vec{\tau}\,)}{\Sigma \vdash (\langle \mathsf{ts\_cons} | \vec{t}\, | t \rangle) : (\langle \mathsf{typs\_cons} | \vec{\tau}\, | \tau \rangle)}\ [\text{tcts\_cons}]$$

1.3 Properties

%mode $\langle \mathsf{form\_eq} | {+\tau_1} | +\tau_2 \rangle$
%mode $+t_1 = +t_2$
%mode $\langle \mathsf{f\_lookup} | {+x} | -\tau | +\Sigma \rangle$
%mode $+\Sigma_1, +\vec{x} : +\vec{\tau} = -\Sigma_2$
%mode

$+\Sigma \vdash +t : -\tau$
$+\Sigma, \langle +\vec{x}\, \rangle : \langle +\vec{\tau}\, \rangle \vdash +t : -\tau'$
$+\Sigma \vdash (+\vec{t}\,) : (-\vec{\tau}\,)$

1.4 Examples

$t_0 \quad = \quad \langle \mathsf{t\_zero} \rangle.$

%solve ⟨f_env_empty⟩⊢t_0: ⟨typ_nat⟩
%solve ⟨f_env_empty⟩⊢⟨t_lam|x|⟨typ_nat⟩|⟨t_succ|⟨t_zero⟩⟩⟩: τ
%solve ⟨f_env_empty⟩⊢⟨t_lam|x|⟨typ_nat⟩|⟨t_succ|⟨t_zero⟩⟩⟩: ⟨typ_imp|⟨typ_nat⟩|⟨typ_nat⟩⟩
%solve ⟨f_env_empty⟩⊢⟨t_lam|x|⟨typ_nat⟩|⟨t_lam|y|⟨typ_nat⟩|⟨t_rec|⟨t_var|x⟩|⟨t_var|y⟩|⟨t_lam|k|⟨typ_nat⟩|⟨t_lam|z|⟨typ_nat⟩|⟨t_succ|⟨t_zero⟩⟩⟩⟩⟩⟩⟩: τ
%solve ⟨f_env_empty⟩⊢⟨t_lam|x|⟨typ_nat⟩|⟨t_var|x⟩⟩: ⟨typ_imp|⟨typ_nat⟩|⟨typ_nat⟩⟩