1 First simulation (soundness)

1.1 Syntax

1.1.1 Index, indices and tables

%datatype *index*
%name *index* n α

$$\begin{array}{lcl} n & ::= & 0 \\ & | & n+1 \end{array}$$

%datatype *vector*
%name *vector* $\mathcal{I}$

$$\begin{array}{lcl} \mathcal{I} & ::= & [\,] \\ & | & n :: \mathcal{I} \end{array}$$

%datatype *table*
%name *table* $\mathcal{I}_\mu$

$$\begin{array}{lcl} \mathcal{I}_\mu & ::= & [\,] \\ & | & \mathcal{I} :: \mathcal{I}_\mu \end{array}$$

1.1.2 Term

%datatype *term*
%name *term* t

$$\begin{array}{lcl} t & ::= & n \\ & | & t_1\, t_2 \\ & | & \lambda t \\ & | & \mathbf{catch}\, t \\ & | & \mathbf{throw}\, \alpha\, t \end{array}$$

Remark. Syntax of safe λ_{ct}-terms:

$$\begin{array}{lcl} t & ::= & n \\ & | & t_1\, t_2 \\ & | & \lambda t \\ & | & \mathbf{get\text{-}context}\, t \\ & | & \mathbf{set\text{-}context}\, \alpha\, t \end{array}$$

1.2 Subtraction

%judgment $n_1 \dot{-} n_2 = n_3$

$$n_1 \dot{-} 0 = n_1 \;[\mathrm{minus}_1]$$

$$(n_1+1) \dot{-} (n_2+1) = n_3 \;[\mathrm{minus}_2] \quad \textit{when} \quad n_1 \dot{-} n_2 = n_3$$

%mode $+n_1 \dot{-} +n_2 = -n_3$

%worlds $()$ $n_1 \dot{-} n_2 = n_3$

%terminates (n_1) $n_1 \dot{-} n_2 = n_3$
%unique $+n_1 \dot{-} +n_2 = -1n_3$

%lemma $\forall n\colon index \cdot n \dot{-} n = 0$ [minus·id]

%lemma $n_1 \dot{-} (n_2 + 1) = n_3 \quad \Rightarrow \quad n_1 \dot{-} n_2 = (n_3 + 1)$ [minus·succ]

%lemma $n_1 \dot{-} n_2 = n_3 \quad \Rightarrow \quad n_1 \dot{-} n_3 = n_2$ [minus·swap]

1.2.1 Fetch (indices)

%judgment $\mathcal{I}(n_1) = n_2$

$$(n :: \mathcal{I})(0) = n \ ^{[\mathrm{fetch}_1^{\mathcal{I}}]}$$
$$(n :: \mathcal{I})(n_1 + 1) = n_2 \ ^{[\mathrm{fetch}_2^{\mathcal{I}}]} \quad when \quad \mathcal{I}(n_1) = n_2$$

%mode $+\mathcal{I}(+n_1) = -n_2$
%worlds $()$ $\mathcal{I}(n_1) = n_2$
%terminates n_1 $\mathcal{I}(n_1) = n_2$
%unique $+\mathcal{I}(+n_1) = -1n_2$

1.2.2 Fetch (table)

%judgment $\mathcal{I}_\mu(n) = \mathcal{I}$

$$(\mathcal{I} :: \mathcal{I}_\mu)(0) = \mathcal{I} \ ^{\left[\mathrm{fetch}_1^{\mathcal{I}_\mu}\right]}$$
$$(\mathcal{I}' :: \mathcal{I}_\mu)(\alpha + 1) = \mathcal{I} \ ^{\left[\mathrm{fetch}_2^{\mathcal{I}_\mu}\right]} \quad when \quad \mathcal{I}_\mu(\alpha) = \mathcal{I}$$

%mode $+\mathcal{I}_\mu(+\alpha) = -\mathcal{I}$
%worlds $()$ $\mathcal{I}_\mu(\alpha) = \mathcal{I}$
%terminates α $\mathcal{I}_\mu(\alpha) = \mathcal{I}$
%unique $+\mathcal{I}_\mu(+\alpha) = -1\mathcal{I}$

1.2.3 Compute

%judgment $n_1 \dot{-} \mathcal{I}(n_2) = n_3$

$$n \dot{-} \mathcal{I}(l) = g \ ^{[\mathrm{compute}_1]} \quad when \quad \mathcal{I}(l) = k, \quad n \dot{-} k = g$$

%mode $+n_1 \dot{-} +\mathcal{I}(+n_2) = -n_3$

%worlds $()$ $n_1 \dot{-} \mathcal{I}(n_2) = n_3$

%terminates $\{\}$ $n_1 \dot{-} \mathcal{I}(n_2) = n_3$

%unique $+n_1 \dot{-} +\mathcal{I}(+n_2) = -1n_3$

2 Safe $\boldsymbol{\lambda}_{\mathrm{ct}}$-terms

2.1 Safety

%judgment $n \in \mathcal{I}$

$$n \in (n :: \mathcal{I}) \qquad ^{[\mathrm{member}_1]}$$
$$n \in (n' :: \mathcal{I}) \qquad ^{[\mathrm{member}_2]} \quad when \quad n \in \mathcal{I}$$

%mode $+n \in +\mathcal{I}$
%worlds $()$ $n \in \mathcal{I}$
%terminates $\mathcal{I}$ $n \in \mathcal{I}$

%lemma $\mathcal{I}(n) = k \quad \Rightarrow \quad k \in \mathcal{I}$ [target]

%judgment $\mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(t)$

$\mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(g)$ [safe1] *when* $n \dot{-} g = k, \quad k \in \mathcal{I}$
$\mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(t\,u)$ [safe2] *when* $\mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(t), \quad \mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(u)$
$\mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(\lambda t)$ [safe3] *when* $\mathit{Safe}_{n+1}^{(n+1 :: \mathcal{I}),\mathcal{I}_\mu}(t)$
$\mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(\mathbf{catch}\, t)$ [safe4] *when* $\mathit{Safe}_n^{\mathcal{I},(\mathcal{I} :: \mathcal{I}_\mu)}(t)$
$\mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(\mathbf{throw}\, \alpha\, t)$ [safe5] *when* $\mathcal{I}_\mu(\alpha) = \mathcal{I}', \quad \mathit{Safe}_n^{\mathcal{I}',\mathcal{I}_\mu}(t)$

%mode $\mathit{Safe}_{+n}^{+\mathcal{I},+\mathcal{I}_\mu}(+t)$
%worlds $()$ $\mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(t)$
%terminates t $\mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(t)$

2.2 From local indices to global indices

%judgment $\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t_1) = t_2$

$\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(l) = g$ [$\downarrow$1] *when* $n \dot{-} \mathcal{I}(l) = g$
$\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t\,u) = t'\,u'$ [$\downarrow$2] *when* $\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t) = t', \quad \downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(u) = u'$
$\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(\lambda t) = \lambda t'$ [$\downarrow$3] *when* $\downarrow_{n+1}^{(n+1 :: \mathcal{I}),\mathcal{I}_\mu}(t) = t'$
$\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(\textbf{get-context}\, t) = \mathbf{catch}\, t'$ [$\downarrow$4] *when* $\downarrow_n^{\mathcal{I},(\mathcal{I} :: \mathcal{I}_\mu)}(t) = t'$
$\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(\textbf{set-context}\, \alpha\, t) = \mathbf{throw}\, \alpha\, t'$ [$\downarrow$5] *when* $\mathcal{I}_\mu(\alpha) = \mathcal{I}', \quad \downarrow_n^{\mathcal{I}',\mathcal{I}_\mu}(t) = t'$

%mode $\downarrow_{+n}^{+\mathcal{I},+\mathcal{I}_\mu}(+t) = -t'$
%worlds $()$ $\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t) = t'$
%terminates t $\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t) = t'$
%unique $\downarrow_{+n}^{+\mathcal{I},+\mathcal{I}_\mu}(+t) = -1t'$

%lemma $\downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t) = t' \quad \Rightarrow \quad \mathit{Safe}_n^{\mathcal{I},\mathcal{I}_\mu}(t')$ [$\downarrow$-safe]

2.3 Examples

$1 \;=\; 0 + 1.$
$2 \;=\; 1 + 1.$
$3 \;=\; 2 + 1.$

%solve _ : $\mathit{Safe}_0^{[],[]}(\lambda \mathbf{catch}\, \lambda(0\,(\mathbf{throw}\, 0\, 1)))$

Remark. This example fails as expected:

%solve _ : $\mathit{Safe}_0^{[],[]}(\lambda\textbf{catch}\,\lambda(1\,(\textbf{throw}\,0\,0)))$

%solve _ : $\downarrow_0^{[],[]}(\lambda\textbf{get-context}\,\lambda(1\,(\textbf{set-context}\,0\,0))) = \lambda\textbf{catch}\,\lambda(1\,(\textbf{throw}\,0\,1))$

%solve d_1 : $\downarrow_0^{[],[]}(\lambda\textbf{get-context}\,\lambda(1\,(\textbf{set-context}\,0\,0))) = \lambda\textbf{catch}\,\lambda(1\,(\textbf{throw}\,0\,1))$
%solve _ : $d_1 \quad \Rightarrow \quad \mathcal{D}_2 \; ^{[\downarrow\cdot\mathrm{safe}]}$

2.4 Closure, environment and stack

%datatype *clos* **%name** *clos* c
%datatype *c-env* **%name** *c-env* $\mathcal{E}$
%datatype *k-env* **%name** *k-env* $\mathcal{E}_\mu$
%datatype *stack* **%name** *stack* $\mathcal{S}$

$$c \quad ::= \quad (t, \mathcal{E}, \mathcal{E}_\mu)$$

$$\begin{array}{lll} \mathcal{E} & ::= & () \\ & | & (c;\mathcal{E}) \end{array}$$

$$\begin{array}{lll} \mathcal{E}_\mu & ::= & () \\ & | & (\mathcal{S};\mathcal{E}_\mu) \end{array}$$

$$\begin{array}{lll} \mathcal{S} & ::= & [] \\ & | & c :: \mathcal{S} \end{array}$$

%datatype *state*
%name *state* σ

$$\sigma \quad ::= \quad \langle t, \mathcal{E}, \mathcal{E}_\mu, \mathcal{S} \rangle$$

2.5 Judgments

2.5.1 Fetch a closure

%judgment $\mathcal{E}(n) = c$

$$(c;\mathcal{E})(0) = c \; ^{[\mathrm{fetch}_1]}$$
$$(c';\mathcal{E})(n+1) = c \; ^{[\mathrm{fetch}_2]} \quad \mathit{when} \quad \mathcal{E}(n) = c$$

%mode $+\mathcal{E}(+n) = -c$
%worlds $()$ $\mathcal{E}(n) = c$
%terminates $\mathcal{E}$ $\mathcal{E}(n) = c$
%unique $+\mathcal{E}(+n) = -1c$

2.5.2 Fetch a stack

%judgment $\mathcal{E}_\mu(n) = \mathcal{S}$

$$(\mathcal{S};\mathcal{E}_\mu)(0) = \mathcal{S} \; ^{[\mathrm{fetch}_1^\mu]}$$
$$(\mathcal{S}';\mathcal{E}_\mu)(n+1) = \mathcal{S} \; ^{[\mathrm{fetch}_2^\mu]} \quad \mathit{when} \quad \mathcal{E}_\mu(n) = \mathcal{S}$$

%mode $+\mathcal{E}_\mu(+n) = -\mathcal{S}$
%worlds $()$ $\mathcal{E}_\mu(n) = \mathcal{S}$
%terminates $\mathcal{E}_\mu$ $\mathcal{E}_\mu(n) = \mathcal{S}$
%unique $+\mathcal{E}_\mu(+n) = -1\mathcal{S}$

2.5.3 Evaluation rules

%judgment $\sigma_1 \rightsquigarrow \sigma_2$

$$\langle k, \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t, \mathcal{E}', \mathcal{E}'_\mu, \mathcal{S}\rangle^{\text{[k·var]}} \quad when \quad \mathcal{E}(k) = (t, \mathcal{E}', \mathcal{E}'_\mu)$$
$$\langle (t\,u), \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t, \mathcal{E}, \mathcal{E}_\mu, (u, \mathcal{E}, \mathcal{E}_\mu) :: \mathcal{S}\rangle^{\text{[k·app]}}$$
$$\langle \lambda t, \mathcal{E}, \mathcal{E}_\mu, c :: \mathcal{S}\rangle \rightsquigarrow \langle t, (c; \mathcal{E}), \mathcal{E}_\mu, \mathcal{S}\rangle^{\text{[k·abs]}}$$
$$\langle \mathbf{catch}\, t, \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t, \mathcal{E}, (\mathcal{S}; \mathcal{E}_\mu), \mathcal{S}\rangle^{\text{[k·catch]}}$$
$$\langle \mathbf{throw}\, \alpha\, t, \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t, \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}'\rangle^{\text{[k·throw]}} \quad when \quad \mathcal{E}_\mu(\alpha) = \mathcal{S}'$$

%mode $+\sigma_1 \rightsquigarrow -\sigma_2$
%worlds $()$ $\sigma_1 \rightsquigarrow \sigma_2$
%unique $+\sigma_1 \rightsquigarrow -1\sigma_2$

2.6 Abstract machine for safe $\boldsymbol{\lambda}_{\mathbf{ct}}$-terms (with indirection tables)

2.6.1 Syntax

%datatype *clos*
%datatype *c-env*
%datatype *k-env*
%datatype *stack*
%name *clos* $\tilde{c}$
%name *c-env* $\tilde{\mathcal{E}}$
%name *k-env* $\tilde{\mathcal{E}}_\mu$
%name *stack* $\tilde{\mathcal{S}}$

$$\tilde{c} ::= \big(t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu\big)$$

$$\begin{aligned} \tilde{\mathcal{E}} &::= () \\ &\;\mid\; (\tilde{c}; \tilde{\mathcal{E}}) \end{aligned}$$

$$\begin{aligned} \tilde{\mathcal{E}}_\mu &::= () \\ &\;\mid\; (\tilde{\mathcal{S}}; \tilde{\mathcal{E}}_\mu) \end{aligned}$$

$$\begin{aligned} \tilde{\mathcal{S}} &::= [] \\ &\;\mid\; \tilde{c} :: \tilde{\mathcal{S}} \end{aligned}$$

%datatype *state*
%name *state* $\tilde{\sigma}$

$$\tilde{\sigma} ::= \langle t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle$$

2.6.2 Fetch a closure

%judgment $\tilde{\mathcal{E}}(n) = \tilde{c}$

$$(\tilde{c}; \tilde{\mathcal{E}})(0) = \tilde{c}^{\,[\text{i·fetch}_1]}$$
$$(\tilde{c}'; \tilde{\mathcal{E}})(n+1) = \tilde{c}^{\,[\text{i·fetch}_2]} \quad when \quad \tilde{\mathcal{E}}(n) = \tilde{c}$$

%mode $+\tilde{\mathcal{E}}(+n) = -\tilde{c}$
%worlds $()$ $\tilde{\mathcal{E}}(n) = \tilde{c}$
%terminates $\tilde{\mathcal{E}}$ $\tilde{\mathcal{E}}(n) = \tilde{c}$

%unique $+\tilde{\mathcal{E}}(+n) = -1\tilde{c}$

2.6.3 Fetch a stack

%judgment $\tilde{\mathcal{E}}_\mu(n) = \tilde{\mathcal{S}}$

$$\big(\tilde{\mathcal{S}}; \tilde{\mathcal{E}}_\mu\big)(0) = \tilde{\mathcal{S}}\ ^{[\text{i}\cdot\text{fetch}_1^\mu]}$$

$$\big(\tilde{\mathcal{S}}'; \tilde{\mathcal{E}}_\mu\big)(n+1) = \tilde{\mathcal{S}}\ ^{[\text{i}\cdot\text{fetch}_2^\mu]} \quad \textit{when} \quad \tilde{\mathcal{E}}_\mu(n) = \tilde{\mathcal{S}}$$

%mode $+\tilde{\mathcal{E}}_\mu(+n) = -\tilde{\mathcal{S}}$
%worlds $()$ $\tilde{\mathcal{E}}_\mu(n) = \tilde{\mathcal{S}}$
%terminates $\tilde{\mathcal{E}}_\mu$ $\tilde{\mathcal{E}}_\mu(n) = \tilde{\mathcal{S}}$
%unique $+\tilde{\mathcal{E}}_\mu(+n) = -1\tilde{\mathcal{S}}$

3 Evaluation rules

%judgment $\tilde{\sigma}_1 \rightsquigarrow \tilde{\sigma}_2$

$$\langle l, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t, n', \mathcal{I}', \mathcal{I}'_\mu, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_\mu, \tilde{\mathcal{S}}\rangle\ ^{[\text{i}\cdot\text{var}]}$$
$$\textit{when} \quad n \dot{-} \mathcal{I}(l) = g, \quad \tilde{\mathcal{E}}(g) = \big(t, n', \mathcal{I}', \mathcal{I}'_\mu, \tilde{\mathcal{E}}', \tilde{\mathcal{E}}'_\mu\big)$$

$$\langle (t\,u), n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \big(u, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu\big) :: \tilde{\mathcal{S}}\rangle\ ^{[\text{i}\cdot\text{app}]}$$

$$\langle \lambda t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{c} :: \tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t, n+1, (n+1 :: \mathcal{I}), \mathcal{I}_\mu, (\tilde{c}; \tilde{\mathcal{E}}), \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle\ ^{[\text{i}\cdot\text{abs}]}$$

$$\langle \textbf{get-context}\, t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t, n, \mathcal{I}, (\mathcal{I} :: \mathcal{I}_\mu), \tilde{\mathcal{E}}, \big(\tilde{\mathcal{S}}; \tilde{\mathcal{E}}_\mu\big), \tilde{\mathcal{S}}\rangle\ ^{[\text{i}\cdot\text{catch}]}$$

$$\langle \textbf{set-context}\, \alpha\, t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t, n, \mathcal{I}', \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}'\rangle\ ^{[\text{i}\cdot\text{throw}]}$$
$$\textit{when} \quad \mathcal{I}_\mu(\alpha) = \mathcal{I}', \quad \tilde{\mathcal{E}}_\mu(\alpha) = \tilde{\mathcal{S}}'$$

%mode $+\sigma_1 \rightsquigarrow -\sigma_2$
%worlds $()$ $\sigma_1 \rightsquigarrow \sigma_2$
%unique $+\sigma_1 \rightsquigarrow -1\sigma_2$

4 Translation

%judgment $\tilde{c}^\star = c$
%judgment $\tilde{\mathcal{S}}^\star = \mathcal{S}$
%judgment $\tilde{\mathcal{E}}^\star = \mathcal{E}$
%judgment $\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu$

$$\big(t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu\big)^\star = (u, \mathcal{E}, \mathcal{E}_\mu)\ ^{[\text{clos}^\star]} \quad \textit{when} \quad \downarrow_n^{\mathcal{I},\mathcal{I}_\mu}(t) = u, \quad \tilde{\mathcal{E}}^\star = \mathcal{E}, \quad \tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu$$

$$[]^\star = []\ ^{[\text{stack}_1^\star]}$$
$$\big(\tilde{c} :: \tilde{\mathcal{S}}\big)^\star = c :: \mathcal{S}\ ^{[\text{stack}_2^\star]} \quad \textit{when} \quad \tilde{c}^\star = c, \quad \tilde{\mathcal{S}}^\star = \mathcal{S}$$

$$()^\star = ()\ ^{[\text{c}\cdot\text{env}_1^\star]}$$
$$(\tilde{c}; \tilde{\mathcal{E}})^\star = (c; \mathcal{E})\ ^{[\text{c}\cdot\text{env}_2^\star]} \quad \textit{when} \quad \tilde{c}^\star = c, \quad \tilde{\mathcal{E}}^\star = \mathcal{E}$$

$$()^\star = ()\ ^{[\text{k}\cdot\text{env}_1^\star]}$$
$$\big(\tilde{\mathcal{S}}; \tilde{\mathcal{E}}_\mu\big)^\star = (\mathcal{S}; \mathcal{E}_\mu)\ ^{[\text{k}\cdot\text{env}_2^\star]} \quad \textit{when} \quad \tilde{\mathcal{S}}^\star = \mathcal{S}, \quad \tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu$$

%mode
$+\,\tilde{c}^\star = -c$
$+\,\tilde{\mathcal{S}}^\star = -\mathcal{S}$
$+\,\tilde{\mathcal{E}}^\star = -\mathcal{E}$
$+\,\tilde{\mathcal{E}}_\mu{}^\star = -\mathcal{E}_\mu$

%worlds ()
$\tilde{c}^\star = c$
$\tilde{\mathcal{S}}^\star = \mathcal{S}$
$\tilde{\mathcal{E}}^\star = \mathcal{E}$
$\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu$

%terminates $\big(\tilde{c}\ \tilde{\mathcal{S}}\ \tilde{\mathcal{E}}\ \tilde{\mathcal{E}}_\mu\big)$
$\tilde{c}^\star = c$
$\tilde{\mathcal{S}}^\star = \mathcal{S}$
$\tilde{\mathcal{E}}^\star = \mathcal{E}$
$\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu$

%unique
$+\,\tilde{c}^\star = -1c$
$+\,\tilde{\mathcal{S}}^\star = -1\mathcal{S}$
$+\,\tilde{\mathcal{E}}^\star = -1\mathcal{E}$
$+\,\tilde{\mathcal{E}}_\mu{}^\star = -1\mathcal{E}_\mu$

%judgment $\tilde{\sigma}^\star = \sigma$
$$\langle t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}}\rangle^\star = \langle u, \mathcal{E}, \mathcal{E}_\mu, \mathcal{S}\rangle \ ^{[\text{state}^\star]} \quad \textit{when} \quad \big(t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu\big)^\star = (u, \mathcal{E}, \mathcal{E}_\mu), \quad \tilde{\mathcal{S}}^\star = \mathcal{S}$$

%mode $+\tilde{\sigma}^\star = -\sigma$
%worlds () $\tilde{\sigma}^\star = \sigma$
%unique $+\tilde{\sigma}^\star = -1\sigma$

5 Soundness

%lemma $\tilde{\mathcal{E}}^\star = \mathcal{E} \quad \wedge \quad \tilde{\mathcal{E}}(n) = \tilde{c} \quad \Rightarrow \quad \tilde{c}^\star = c \quad \wedge \quad \mathcal{E}(n) = c \quad \textit{for some } c \quad ^{[\text{fetch}\cdot\text{sound}]}$

%lemma $\tilde{\mathcal{E}}_\mu{}^\star = \mathcal{E}_\mu \quad \wedge \quad \tilde{\mathcal{E}}_\mu(\alpha) = \tilde{\mathcal{S}} \quad \Rightarrow \quad \tilde{\mathcal{S}}^\star = \mathcal{S} \quad \wedge \quad \mathcal{E}_\mu(\alpha) = \mathcal{S} \quad \textit{for some } \mathcal{S} \quad ^{[\text{fetch}^\mu\cdot\text{sound}]}$

%theorem $\tilde{\sigma}_1 \rightsquigarrow \tilde{\sigma}_2 \quad \wedge \quad \tilde{\sigma}_1{}^\star = \sigma_1 \quad \Rightarrow \quad \sigma_1 \rightsquigarrow \sigma_2 \quad \wedge \quad \tilde{\sigma}_2{}^\star = \sigma_2 \quad \textit{for some } \sigma_2 \quad ^{[\text{soundness}]}$