1 Second simulation (soundness)

1.1 Syntax

1.1.1 Index, indices and tables

%datatype *index*
%name *index* n α

$$\begin{array}{lcl} n & ::= & 0 \\ & | & n+1 \end{array}$$

%datatype *vector*
%name *vector* $\mathcal{I}$

$$\begin{array}{lcl} \mathcal{I} & ::= & [] \\ & | & n :: \mathcal{I} \end{array}$$

%datatype *table*
%name *table* $\mathcal{I}_\mu$

$$\begin{array}{lcl} \mathcal{I}_\mu & ::= & [] \\ & | & \mathcal{I} :: \mathcal{I}_\mu \end{array}$$

1.1.2 Term

%datatype *term*
%name *term* t

$$\begin{array}{lcl} t & ::= & n \\ & | & t_1\, t_2 \\ & | & \lambda t \\ & | & \textbf{get-context}\, t \\ & | & \textbf{set-context}\, \alpha\, t \end{array}$$

1.2 Subtraction

%judgment $n_1 \dot{-} n_2 = n_3$

$n_1 \dot{-} 0 = n_1$ [minus$_1$]

$(n_1+1) \dot{-} (n_2+1) = n_3$ [minus$_2$] *when* $n_1 \dot{-} n_2 = n_3$

%mode $+n_1 \dot{-} +n_2 = -n_3$
%worlds $()$ $n_1 \dot{-} n_2 = n_3$
%terminates (n_1) $n_1 \dot{-} n_2 = n_3$
%unique $+n_1 \dot{-} +n_2 = -1n_3$

%lemma $\forall n \cdot n \dot{-} n = 0$ [minus·equals]

1.2.1 Fetch (indices)

%judgment $\mathcal{I}(n_1) = n_2$

$$(n{::}\mathcal{I})(0)=n^{\;[\mathrm{fetch}_1^{\mathcal{I}}]}$$
$$(n{::}\mathcal{I})(n_1+1)=n_2^{\;[\mathrm{fetch}_2^{\mathcal{I}}]}\quad \mathit{when}\quad \mathcal{I}(n_1)=n_2$$

%mode $+\mathcal{I}(+n_1)=-n_2$
%worlds $()\quad \mathcal{I}(n_1)=n_2$
%terminates $n_1\quad \mathcal{I}(n_1)=n_2$
%unique $+\mathcal{I}(+n_1)=-1n_2$

1.2.2 Fetch (table)

%judgment $\mathcal{I}_\mu(n)=\mathcal{I}$

$$(\mathcal{I}::\mathcal{I}_\mu)(0)=\mathcal{I}^{\left[\mathrm{fetch}_1^{\mathcal{I}_\mu}\right]}$$
$$(\mathcal{I}'::\mathcal{I}_\mu)(\alpha+1)=\mathcal{I}^{\left[\mathrm{fetch}_2^{\mathcal{I}_\mu}\right]}\quad \mathit{when}\quad \mathcal{I}_\mu(\alpha)=\mathcal{I}$$

%mode $+\mathcal{I}_\mu(+\alpha)=-\mathcal{I}$
%worlds $()\quad \mathcal{I}_\mu(\alpha)=\mathcal{I}$
%terminates $\alpha\quad \mathcal{I}_\mu(\alpha)=\mathcal{I}$
%unique $+\mathcal{I}_\mu(+\alpha)=-1\mathcal{I}$

1.2.3 Compute

%judgment $n_1\dot{-}\mathcal{I}(n_2)=n_3$

$$n\dot{-}\mathcal{I}(l)=g^{\;[\mathrm{compute}_1]}\quad \mathit{when}\quad \mathcal{I}(l)=k,\quad n\dot{-}k=g$$

%mode $+n_1\dot{-}+\mathcal{I}(+n_2)=-n_3$

%worlds $()\quad n_1\dot{-}\mathcal{I}(n_2)=n_3$

%terminates $\{\}\quad n_1\dot{-}\mathcal{I}(n_2)=n_3$

%unique $+n_1\dot{-}+\mathcal{I}(+n_2)=-1n_3$

1.2.4 Closure, environment and stack

%datatype *clos* **%name** *clos* c
%datatype *l-env* **%name** *l-env* $\mathcal{L}$
%datatype *l-table* **%name** *l-table* $\mathcal{L}_\mu$
%datatype *k-env* **%name** *k-env* $\mathcal{E}_\mu$
%datatype *stack* **%name** *stack* $\mathcal{S}$

$$\begin{array}{lcl}
c & ::= & (t,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu)\\[1ex]
\mathcal{L} & ::= & ()\\
 & \mid & (c;\mathcal{L})\\[1ex]
\mathcal{L}_\mu & ::= & ()\\
 & \mid & \mathcal{L};\mathcal{L}_\mu\\[1ex]
\mathcal{E}_\mu & ::= & ()\\
 & \mid & (\mathcal{S};\mathcal{E}_\mu)\\[1ex]
\mathcal{S} & ::= & []\\
 & \mid & c::\mathcal{S}
\end{array}$$

%datatype *state*

%name *state* σ

$$\sigma \quad ::= \quad \langle t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}\rangle$$

1.3 Judgments

1.3.1 Fetch a local closure

%judgment $\mathcal{L}(n) = c$

$$(c; \mathcal{L})(0) = c^{\,[\mathrm{fetch}_1]}$$
$$(c'; \mathcal{L})(n+1) = c^{\,[\mathrm{fetch}_2]} \quad \textit{when} \quad \mathcal{L}(n) = c$$

%mode $+\mathcal{L}(+n) = -c$
%worlds $()$ $\mathcal{L}(n) = c$
%terminates $\mathcal{L}$ $\mathcal{L}(n) = c$
%unique $+\mathcal{L}(+n) = -1c$

1.3.2 Fetch a local environment

%judgment $\mathcal{L}_\mu(n) = \mathcal{L}$

$$(\mathcal{L}; \mathcal{L}_\mu)(0) = \mathcal{L}^{\,[\mathrm{l{\cdot}fetch}_1]}$$
$$(\mathcal{L}'; \mathcal{L}_\mu)(n+1) = \mathcal{L}^{\,[\mathrm{l{\cdot}fetch}_2]} \quad \textit{when} \quad \mathcal{L}_\mu(n) = \mathcal{L}$$

%mode $+\mathcal{L}_\mu(+n) = -\mathcal{L}$
%worlds $()$ $\mathcal{L}_\mu(n) = \mathcal{L}$
%terminates $\mathcal{L}_\mu$ $\mathcal{L}_\mu(n) = \mathcal{L}$
%unique $+\mathcal{L}_\mu(+n) = -1\mathcal{L}$

1.3.3 Fetch a stack

%judgment $\mathcal{E}_\mu(n) = \mathcal{S}$

$$(\mathcal{S}; \mathcal{E}_\mu)(0) = \mathcal{S}^{\,[\mathrm{fetch}^\mu_1]}$$
$$(\mathcal{S}'; \mathcal{E}_\mu)(n+1) = \mathcal{S}^{\,[\mathrm{fetch}^\mu_2]} \quad \textit{when} \quad \mathcal{E}_\mu(n) = \mathcal{S}$$

%mode $+\mathcal{E}_\mu(+n) = -\mathcal{S}$
%worlds $()$ $\mathcal{E}_\mu(n) = \mathcal{S}$
%terminates $\mathcal{E}_\mu$ $\mathcal{E}_\mu(n) = \mathcal{S}$
%unique $+\mathcal{E}_\mu(+n) = -1\mathcal{S}$

1.3.4 Evaluation rules

%judgment $\sigma_1 \rightsquigarrow \sigma_2$

$$\langle k, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t, \mathcal{L}', \mathcal{L}'_\mu, \mathcal{E}'_\mu, \mathcal{S}\rangle^{\,[\mathrm{k{\cdot}var}]} \quad \textit{when} \quad \mathcal{L}(k) = (t, \mathcal{L}', \mathcal{L}'_\mu, \mathcal{E}'_\mu)$$
$$\langle (t\,u), \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, (u, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu) :: \mathcal{S}\rangle^{\,[\mathrm{k{\cdot}app}]}$$
$$\langle \lambda t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, c :: \mathcal{S}\rangle \rightsquigarrow \langle t, (c; \mathcal{L}), \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}\rangle^{\,[\mathrm{k{\cdot}abs}]}$$
$$\langle \textbf{get-context}\, t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t, \mathcal{L}, (\mathcal{L}; \mathcal{L}_\mu), (\mathcal{S}; \mathcal{E}_\mu), \mathcal{S}\rangle^{\,[\mathrm{k{\cdot}catch}]}$$
$$\langle \textbf{set-context}\, \alpha\, t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t, \mathcal{L}', \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}'\rangle^{\,[\mathrm{k{\cdot}throw}]} \quad \textit{when} \quad \mathcal{L}_\mu(\alpha) = \mathcal{L}', \quad \mathcal{E}_\mu(\alpha) = \mathcal{S}'$$

%mode $+\sigma_1 \rightsquigarrow -\sigma_2$
%worlds $()$ $\sigma_1 \rightsquigarrow \sigma_2$
%unique $+\sigma_1 \rightsquigarrow -1\sigma_2$

1.4 Abstract machine for safe $\boldsymbol{\lambda}_{\mathbf{ct}}$-terms

1.4.1 Syntax

%datatype *clos*
%datatype *c-env*
%datatype *k-env*
%datatype *stack*
%name *clos* $\tilde{c}$
%name *c-env* $\tilde{\mathcal{E}}$
%name *k-env* $\tilde{\mathcal{E}}_\mu$
%name *stack* $\tilde{\mathcal{S}}$

$$\begin{array}{lcl} \tilde{c} & ::= & \big(t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu\big) \\[1ex] \tilde{\mathcal{E}} & ::= & () \\ & | & (\tilde{c}; \tilde{\mathcal{E}}) \\[1ex] \tilde{\mathcal{E}}_\mu & ::= & () \\ & | & (\tilde{\mathcal{S}}; \tilde{\mathcal{E}}_\mu) \\[1ex] \tilde{\mathcal{S}} & ::= & [] \\ & | & \tilde{c} :: \tilde{\mathcal{S}} \end{array}$$

%datatype *state*
%name *state* $\tilde{\sigma}$

$$\tilde{\sigma} \quad ::= \quad \langle t, n, \mathcal{I}, \mathcal{I}_\mu, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_\mu, \tilde{\mathcal{S}} \rangle$$

1.4.2 Fetch a closure

%judgment $\tilde{\mathcal{E}}(n) = \tilde{c}$

$$(\tilde{c}; \tilde{\mathcal{E}})(0) = \tilde{c}^{\,[\text{i}\cdot\text{fetch}_1]}$$
$$(\tilde{c}'; \tilde{\mathcal{E}})(n+1) = \tilde{c}^{\,[\text{i}\cdot\text{fetch}_2]} \quad \textit{when} \quad \tilde{\mathcal{E}}(n) = \tilde{c}$$

%mode $+\tilde{\mathcal{E}}(+n) = -\tilde{c}$
%worlds $()$ $\tilde{\mathcal{E}}(n) = \tilde{c}$
%terminates $\tilde{\mathcal{E}}$ $\tilde{\mathcal{E}}(n) = \tilde{c}$
%unique $+\tilde{\mathcal{E}}(+n) = -1\tilde{c}$

1.4.3 Compute

%judgment $\tilde{\mathcal{E}}(n_1 \dot{-} n_2) = \tilde{c}$

$$\tilde{\mathcal{E}}(n \dot{-} k) = \tilde{c}^{\,[\text{i}\cdot\text{compute}_1]} \quad \textit{when} \quad n \dot{-} k = g, \quad \tilde{\mathcal{E}}(g) = \tilde{c}$$

%mode $+\tilde{\mathcal{E}}(+n \dot{-} +k) = -\tilde{c}$
%worlds $()$ $\tilde{\mathcal{E}}(n \dot{-} k) = \tilde{c}$
%terminates $\{\}$ $\tilde{\mathcal{E}}(n \dot{-} k) = \tilde{c}$
%unique $+\tilde{\mathcal{E}}(+n \dot{-} +k) = -1\tilde{c}$

1.4.4 Fetch a stack

%judgment $\tilde{\mathcal{E}}_\mu(n) = \tilde{\mathcal{S}}$

$(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu)(0)=\tilde{\mathcal{S}}$ [i·fetch$_1^\mu$]
$(\tilde{\mathcal{S}}';\tilde{\mathcal{E}}_\mu)(n+1)=\tilde{\mathcal{S}}$ [i·fetch$_2^\mu$] *when* $\tilde{\mathcal{E}}_\mu(n)=\tilde{\mathcal{S}}$

%mode $+\tilde{\mathcal{E}}_\mu(+n)=-\tilde{\mathcal{S}}$
%worlds $()$ $\tilde{\mathcal{E}}_\mu(n)=\tilde{\mathcal{S}}$
%terminates $\tilde{\mathcal{E}}_\mu$ $\tilde{\mathcal{E}}_\mu(n)=\tilde{\mathcal{S}}$
%unique $+\tilde{\mathcal{E}}_\mu(+n)=-1\tilde{\mathcal{S}}$

2 Evaluation rules

%judgment $\tilde{\sigma}_1 \rightsquigarrow \tilde{\sigma}_2$

$\langle l,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t,n',\mathcal{I}',\mathcal{I}'_\mu,\tilde{\mathcal{E}}',\tilde{\mathcal{E}}'_\mu,\tilde{\mathcal{S}}\rangle$ [i·var]
when $n\dot{-}\mathcal{I}(l)=g,\quad \tilde{\mathcal{E}}(g)=\big(t,n',\mathcal{I}',\mathcal{I}'_\mu,\tilde{\mathcal{E}}',\tilde{\mathcal{E}}'_\mu\big)$

$\langle (t\,u),n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\big(u,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu\big)::\tilde{\mathcal{S}}\rangle$ [i·app]

$\langle \lambda t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{c}::\tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t,n+1,(n+1::\mathcal{I}),\mathcal{I}_\mu,(\tilde{c};\tilde{\mathcal{E}}),\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle$ [i·abs]

$\langle \textbf{get-context}\,t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t,n,\mathcal{I},(\mathcal{I}::\mathcal{I}_\mu),\tilde{\mathcal{E}},(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu),\tilde{\mathcal{S}}\rangle$ [i·catch]

$\langle \textbf{set-context}\,\alpha\,t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}\rangle \rightsquigarrow \langle t,n,\mathcal{I}',\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu,\tilde{\mathcal{S}}'\rangle$ [i·throw]
when $\mathcal{I}_\mu(\alpha)=\mathcal{I}',\quad \tilde{\mathcal{E}}_\mu(\alpha)=\tilde{\mathcal{S}}'$

%mode $+\sigma_1 \rightsquigarrow -\sigma_2$
%worlds $()$ $\sigma_1 \rightsquigarrow \sigma_2$
%unique $+\sigma_1 \rightsquigarrow -1\sigma_2$

3 Translation

%judgment $\tilde{c}^\diamond=c$
%judgment $\tilde{\mathcal{S}}^\diamond=\mathcal{S}$
%judgment $\tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu$
%judgment $\mathit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I}=\mathcal{L}$
%judgment $\mathit{map}\ \big(\mathit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ \mathcal{I}_\mu=\mathcal{L}_\mu$

$\big(t,n,\mathcal{I},\mathcal{I}_\mu,\tilde{\mathcal{E}},\tilde{\mathcal{E}}_\mu\big)^\diamond=(t,\mathcal{L},\mathcal{L}_\mu,\mathcal{E}_\mu)$ [clos$^\diamond$] *when* $\mathit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I}=\mathcal{L},\quad \mathit{map}\ \big(\mathit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ \mathcal{I}_\mu=\mathcal{L}_\mu,\quad \tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu$

$[]^\diamond=[]$ [stack$_1^\diamond$]
$\big(\tilde{c}::\tilde{\mathcal{S}}\big)^\diamond=c::\mathcal{S}$ [stack$_2^\diamond$] *when* $\tilde{c}^\diamond=c,\quad \tilde{\mathcal{S}}^\diamond=\mathcal{S}$

$()^\diamond=()$ [k·env$_1^\diamond$]
$(\tilde{\mathcal{S}};\tilde{\mathcal{E}}_\mu)^\diamond=(\mathcal{S};\mathcal{E}_\mu)$ [k·env$_2^\diamond$] *when* $\tilde{\mathcal{S}}^\diamond=\mathcal{S},\quad \tilde{\mathcal{E}}_\mu{}^\diamond=\mathcal{E}_\mu$

$\mathit{flatten}\ n\ \tilde{\mathcal{E}}\ []=()$ [flatten$_1$]
$\mathit{flatten}\ n\ \tilde{\mathcal{E}}\ (k::\mathcal{I})=(c;\mathcal{L})$ [flatten$_2$] *when* $\tilde{\mathcal{E}}(n\dot{-}k)=\tilde{c},\quad \tilde{c}^\diamond=c,\quad \mathit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I}=\mathcal{L}$

$\mathit{map}\ \big(\mathit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ []=()$ [map$_1$]
$\mathit{map}\ \big(\mathit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ (\mathcal{I}::\mathcal{I}_\mu)=\mathcal{L};\mathcal{L}_\mu$ [map$_2$] *when* $\mathit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I}=\mathcal{L},\quad \mathit{map}\ \big(\mathit{flatten}\ n\ \tilde{\mathcal{E}}\big)\ \mathcal{I}_\mu=\mathcal{L}_\mu$

%mode

$+\tilde{c}^{\diamond} = -c$
$+\tilde{\mathcal{S}}^{\diamond} = -\mathcal{S}$
$+\tilde{\mathcal{E}_{\mu}}^{\diamond} = -\mathcal{E}_{\mu}$
$\mathit{flatten}\ +n\ +\tilde{\mathcal{E}}\ +\mathcal{I} = -\mathcal{L}$
$\mathit{map}\ (\mathit{flatten}\ +n\ +\tilde{\mathcal{E}})\ +\mathcal{I}_{\mu} = -\mathcal{L}_{\mu}$

%worlds ()

$\tilde{c}^{\diamond} = c$
$\tilde{\mathcal{S}}^{\diamond} = \mathcal{S}$
$\tilde{\mathcal{E}_{\mu}}^{\diamond} = \mathcal{E}_{\mu}$
$\mathit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}$
$\mathit{map}\ (\mathit{flatten}\ n\ \tilde{\mathcal{E}})\ \mathcal{I}_{\mu} = \mathcal{L}_{\mu}$

Remark. To do.

%terminates $(\tilde{c}\ \tilde{\mathcal{S}}\ \tilde{\mathcal{E}}_{\mu}\ \mathcal{I}\ \mathcal{I}_{\mu})$

$\tilde{c}^{\diamond} = c$
$\tilde{\mathcal{S}}^{\diamond} = \mathcal{S}$
$\tilde{\mathcal{E}_{\mu}}^{\diamond} = \mathcal{E}_{\mu}$
$\mathit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}$
$\mathit{map}\ (\mathit{flatten}\ n'\ \tilde{\mathcal{E}}')\ \mathcal{I}_{\mu} = \mathcal{L}_{\mu}$

%unique

$+\tilde{c}^{\diamond} = -1c$
$+\tilde{\mathcal{S}}^{\diamond} = -1\mathcal{S}$
$+\tilde{\mathcal{E}_{\mu}}^{\diamond} = -1\mathcal{E}_{\mu}$
$\mathit{flatten}\ +n\ +\tilde{\mathcal{E}}\ +\mathcal{I} = -1\mathcal{L}$
$\mathit{map}\ (\mathit{flatten}\ +n\ +\tilde{\mathcal{E}})\ +\mathcal{I}_{\mu} = -1\mathcal{L}_{\mu}$

%judgment $\tilde{\sigma}^{\diamond} = \sigma$

$\langle t, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}, \tilde{\mathcal{S}}\rangle^{\diamond} = \langle t', \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu}, \mathcal{S}\rangle$ [state◇] *when* $(t, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu})^{\diamond} = (t', \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu}), \quad \tilde{\mathcal{S}}^{\diamond} = \mathcal{S}$

%mode $+\tilde{\sigma}^{\diamond} = -\sigma$
%worlds () $\tilde{\sigma}^{\diamond} = \sigma$
%unique $+\tilde{\sigma}^{\diamond} = -1\sigma$

4 Soundness

%lemma $n \dot{-} k = g \quad \Rightarrow \quad (n+1) \dot{-} k = g+1$ [minus·succ]

%lemma $\forall \tilde{c}' \cdot \mathit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L} \quad \Rightarrow \quad \mathit{flatten}\ (n+1)\ (\tilde{c}'; \tilde{\mathcal{E}})\ \mathcal{I} = \mathcal{L}$ [weaken·flatten]

%lemma $\forall \tilde{c}' \cdot \mathit{map}\ (\mathit{flatten}\ n\ \tilde{\mathcal{E}})\ \mathcal{I}_{\mu} = \mathcal{L}_{\mu} \quad \Rightarrow \quad \mathit{map}\ (\mathit{flatten}\ (n+1)\ (\tilde{c}'; \tilde{\mathcal{E}}))\ \mathcal{I}_{\mu} = \mathcal{L}_{\mu}$ [weaken·map]

%lemma $\mathit{map}\ (\mathit{flatten}\ n\ \tilde{\mathcal{E}})\ \mathcal{I}_{\mu} = \mathcal{L}_{\mu} \quad \wedge \quad \mathcal{I}_{\mu}(\alpha) = \mathcal{I}' \quad \Rightarrow \quad \mathit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I}' = \mathcal{L}' \quad \wedge \quad \mathcal{L}_{\mu}(\alpha) = \mathcal{L}'$ [map·sound]

%lemma $\mathcal{I}(l) = k \quad \wedge \quad \tilde{\mathcal{E}}(n \dot{-} k) = \tilde{c} \quad \wedge \quad \mathit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L} \quad \Rightarrow \quad \tilde{c}^{\diamond} = c \quad \wedge \quad \mathcal{L}(l) = c$ [fetch·sound]

%lemma $\tilde{\mathcal{E}}_\mu{}^\diamond = \mathcal{E}_\mu \quad \wedge \quad \tilde{\mathcal{E}}_\mu(\alpha) = \tilde{\mathcal{S}} \quad \Rightarrow \quad \tilde{\mathcal{S}}^\diamond = \mathcal{S} \quad \wedge \quad \mathcal{E}_\mu(\alpha) = \mathcal{S}$ [fetch$^\mu$·sound]

%theorem $\tilde{\sigma}_1 \rightsquigarrow \tilde{\sigma}_2 \quad \wedge \quad \tilde{\sigma}_1{}^\diamond = \sigma_1 \quad \Rightarrow \quad \sigma_1 \rightsquigarrow \sigma_2 \quad \wedge \quad \tilde{\sigma}_2{}^\diamond = \sigma_2$ [soundness]