# 1 Krivine abstract machine (for De Bruijn encoding)

## 1.1 Syntax

*unary* : **type**.
*term* : **type**.
*clos* : **type**.
*env* : **type**.
*stack* : **type**.
*state* : **type**.

**Unary numbers $n$**

$0$ : *unary*.
$n+1$ : *unary*.

**Term $t$**

$n$ : *term*.
$t_1\, t_2$ : *term*.
$\lambda t$ : *term*.
$\lambda x.t$ : *term*.

**binding** $1 \mapsto 2$ **in** $\lambda \sqcup . \sqcup$

**Closure $c$**

$(t, \mathcal{E})$ : *clos*.

**Environment $\mathcal{E}$**

$[]$ : *env*.
$(\mathcal{E}, c)$ : *env*.

**Stack $\mathcal{S}$**

$[]$ : *stack*.
$c : \mathcal{S}$ : *stack*.

**State $\sigma$**

$\langle t, \mathcal{E}, \mathcal{S} \rangle$ : *state*.

## 1.2 Judgments

$\mathcal{E}(n) = c$ : **type**.
$\sigma_1 \to \sigma_2$ : **type**.
$[x \leftarrow n]t_1 = t_2$ : **type**.

**binding** $1 \mapsto 3$ **in** $[\sqcup \leftarrow \sqcup]\sqcup = \sqcup$

## 1.3 Meta substitution

$$[x \leftarrow n]x = n \ \text{[subst\_var]}$$

$$[x \leftarrow n]t = t \ \text{[subst\_term]}$$

$$\frac{[x \leftarrow n](t_1[x]) = t_1' \quad [x \leftarrow n](t_2[x]) = t_2'}{[x \leftarrow n](t_1[x]\ t_2[x]) = t_1'\ t_2'} \ \text{[subst\_app]}$$

$$\frac{[x \leftarrow n+1](t[x]) = t'}{[x \leftarrow n](\lambda t[x]) = \lambda t'} \ \text{[subst\_lam\_1]}$$

$$\frac{\mathbf{\{}y\mathbf{\}} \ [x \leftarrow n+1](t[y][x]) = t'[y]}{[x \leftarrow n](\lambda y.t[y][x]) = \lambda y.t'[y]} \ \text{[subst\_lam\_2]}$$

**%solve** $[x \leftarrow 0]\lambda y.x\, y = \sqcup$

**%block** *ind_block* : **block** $\{x\colon \mathit{term}\}$
**%mode** $[\sqcup \leftarrow +n_1] + t_2 = -t_3$
**%worlds** $(\mathit{ind\_block})$ $[\sqcup \leftarrow n_1]t_2 = t_3$
**%terminates** $(t_2)$ $[\sqcup \leftarrow n_1]t_2 = t_3$

**Remark.** Not checked by Twelf.
**%unique** $[\sqcup \leftarrow +n_1] + t_2 = -1t_3$

## 1.4 Fetch

$$\frac{}{(\mathcal{E}, c)(0) = c} \ \text{[fetch\_1]}$$

$$\frac{\mathcal{E}(n) = c}{(\mathcal{E}, c')(n+1) = c} \ \text{[fetch\_2]}$$

**%mode** $+\mathcal{E}(+n) = -c$
**%worlds** $()$ $\mathcal{E}(n) = c$
**%terminates** $\mathcal{E}$ $\mathcal{E}(n) = c$
**%unique** $+\mathcal{E}(+n) = -1c$

## 1.5 Evaluation

$$\frac{\mathcal{E}(n) = (t, \mathcal{E}')}{\langle n, \mathcal{E}, \mathcal{S}\rangle \rightarrow \langle t, \mathcal{E}', \mathcal{S}\rangle} \ \text{[step\_var]}$$

$$\frac{}{\langle (t_1\, t_2), \mathcal{E}, \mathcal{S}\rangle \rightarrow \langle t_1, \mathcal{E}, (t_2, \mathcal{E}) : \mathcal{S}\rangle} \ \text{[step\_app]}$$

$$\frac{}{\langle \lambda t, \mathcal{E}, c : \mathcal{S}\rangle \rightarrow \langle t, (\mathcal{E}, c), \mathcal{S}\rangle} \ \text{[step\_lam\_1]}$$

$$\frac{[x \leftarrow 0]t = t'}{\langle \lambda x.t, \mathcal{E}, c : \mathcal{S}\rangle \rightarrow \langle \lambda t', \mathcal{E}, c : \mathcal{S}\rangle} \ \text{[step\_lam\_2]}$$

**%mode** $+\sigma_1 \rightarrow -\sigma_2$
**%worlds** $()$ $\sigma_1 \rightarrow \sigma_2$

**Remark.**
**%unique** $+\sigma_1 \rightarrow -1\sigma_2$