# Modified machine: local environments

**%datatype** *clos*
**%datatype** *l-env*
**%datatype** *l-table*
**%datatype** *k-env*
**%datatype** *stack*

**%name** *clos* $c$
**%name** *l-env* $\mathcal{L}$
**%name** *l-table* $\mathcal{L}_\mu$
**%name** *k-env* $\mathcal{E}_\mu$
**%name** *stack* $\mathcal{S}$

$$
\begin{array}{lcl}
c & ::= & (t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu) \\
\\
\mathcal{L} & ::= & () \\
 & | & (c; \mathcal{L}) \\
\\
\mathcal{L}_\mu & ::= & () \\
 & | & (\mathcal{L}; \mathcal{L}_\mu)
\end{array}
$$

$$
\begin{array}{lcl}
\mathcal{E}_\mu & ::= & () \\
 & | & (\mathcal{S}; \mathcal{E}_\mu) \\
\\
\mathcal{S} & ::= & [] \\
 & | & c :: \mathcal{S}
\end{array}
$$

**%datatype** *state*

**%name** *state* $\sigma$

$$
\sigma \quad ::= \quad \langle t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S} \rangle
$$

# Modified machine: evaluation rules

**%judgment** $\sigma_1 \rightsquigarrow \sigma_2$

$$\langle n, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t, \mathcal{L}', \mathcal{L}'_\mu, \mathcal{E}'_\mu, \mathcal{S}\rangle \qquad \text{[k·var]}$$
$$\quad \textit{when} \quad \mathcal{L}(n) = (t, \mathcal{L}', \mathcal{L}'_\mu, \mathcal{E}'_\mu)$$

$$\langle (t\,u), \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, (u, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu) :: \mathcal{S}\rangle \qquad \text{[k·app]}$$

$$\langle \lambda t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, c :: \mathcal{S}\rangle \rightsquigarrow \langle t, (c; \mathcal{L}), \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}\rangle \qquad \text{[k·abs]}$$

$$\langle \textbf{get-context}\, t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t, \mathcal{L}, (\mathcal{L}; \mathcal{L}_\mu), (\mathcal{S}; \mathcal{E}_\mu), \mathcal{S}\rangle \qquad \text{[k·catch]}$$

$$\langle \textbf{set-context}\, \alpha\, t, \mathcal{L}, \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}\rangle \rightsquigarrow \langle t, \mathcal{L}', \mathcal{L}_\mu, \mathcal{E}_\mu, \mathcal{S}'\rangle \qquad \text{[k·throw]}$$
$$\quad \textit{when} \quad \mathcal{L}_\mu(\alpha) = \mathcal{L}', \quad \mathcal{E}_\mu(\alpha) = \mathcal{S}'$$

**%unique** $+\sigma_1 \rightsquigarrow -1\sigma_2$

# Translation $(\text{-})^{\diamond}$

**%judgment** $\tilde{c}^{\diamond} = c$
**%judgment** $\tilde{\mathcal{S}}^{\diamond} = \mathcal{S}$
**%judgment** $\tilde{\mathcal{E}}_{\mu}{}^{\diamond} = \mathcal{E}_{\mu}$
**%judgment** $\mathit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}$
**%judgment** $\mathit{map}\ \left(\mathit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ \mathcal{I}_{\mu} = \mathcal{L}_{\mu}$

$$\left(t, n, \mathcal{I}, \mathcal{I}_{\mu}, \tilde{\mathcal{E}}, \tilde{\mathcal{E}}_{\mu}\right)^{\diamond} = (t, \mathcal{L}, \mathcal{L}_{\mu}, \mathcal{E}_{\mu})\ ^{[\mathrm{clos}^{\diamond}]}$$
$$\quad \mathit{when}\quad \mathit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}, \quad \mathit{map}\ \left(\mathit{flatten}\ n\ \tilde{\mathcal{E}}\right)\ \mathcal{I}_{\mu} = \mathcal{L}_{\mu}, \quad \tilde{\mathcal{E}}_{\mu}{}^{\diamond} = \mathcal{E}_{\mu}$$

$$\mathit{flatten}\ n\ \tilde{\mathcal{E}}\ \ [\,] = ()\ ^{[\mathrm{flatten}_1]}$$
$$\mathit{flatten}\ n\ \tilde{\mathcal{E}}\ \ (k{::}\mathcal{I}) = (c; \mathcal{L})\ ^{[\mathrm{flatten}_2]}$$
$$\quad \mathit{when}\quad \tilde{\mathcal{E}}(n \dot{-} k) = \tilde{c}, \quad \tilde{c}^{\diamond} = c, \quad \mathit{flatten}\ n\ \tilde{\mathcal{E}}\ \mathcal{I} = \mathcal{L}$$

The definitions of the remaining judgments are compositional.

# Functional bi-simulations

## Translation $(\text{-})^{\star}$

**%theorem**

$$\tilde{\sigma}_1 \rightsquigarrow \tilde{\sigma}_2 \quad \wedge \quad \tilde{\sigma}_1^{\star} = \sigma_1 \quad \Rightarrow \quad \sigma_1 \rightsquigarrow \sigma_2 \quad \wedge \quad \tilde{\sigma}_2^{\star} = \sigma_2 \quad \textit{for some } \sigma_2 \qquad \text{[soundness]}$$

**%theorem**

$$\sigma_1 \rightsquigarrow \sigma_2 \quad \wedge \quad \tilde{\sigma}_1^{\star} = \sigma_1 \quad \Rightarrow \quad \tilde{\sigma}_1 \rightsquigarrow \tilde{\sigma}_2 \quad \wedge \quad \tilde{\sigma}_2^{\star} = \sigma_2 \quad \textit{for some } \tilde{\sigma}_2 \qquad \text{[completeness]}$$

## Translation $(\text{-})^{\diamond}$

**%theorem**

$$\tilde{\sigma}_1 \rightsquigarrow \tilde{\sigma}_2 \quad \wedge \quad \tilde{\sigma}_1{}^{\diamond} = \sigma_1 \quad \Rightarrow \quad \sigma_1 \rightsquigarrow \sigma_2 \quad \wedge \quad \tilde{\sigma}_2{}^{\diamond} = \sigma_2 \quad \textit{for some } \sigma_2 \qquad \text{[soundness]}$$

**%theorem**

$$\sigma_1 \rightsquigarrow \sigma_2 \quad \wedge \quad \tilde{\sigma}_1^{\diamond} = \sigma_1 \quad \Rightarrow \quad \tilde{\sigma}_1 \rightsquigarrow \tilde{\sigma}_2 \quad \wedge \quad \tilde{\sigma}_2^{\diamond} = \sigma_2 \quad \textit{for some } \tilde{\sigma}_2 \qquad \text{[completeness]}$$