Règle de deductions naturelles

$$\frac{}{\Gamma, \phi \vdash \phi} \; Ax$$

$$\frac{\Gamma, \vdash \phi \quad \Gamma \vdash \psi}{\Gamma \vdash \phi \wedge \psi} \; \wedge_i \qquad\qquad \frac{\Gamma \vdash \phi \wedge \psi}{\Gamma \vdash \phi} \; \wedge_{e1} \qquad \frac{\Gamma \vdash \phi \wedge \psi}{\Gamma \vdash \psi} \; \wedge_{e2}$$

$$\frac{\Gamma, \phi \vdash \psi}{\Gamma \vdash \phi \Rightarrow \psi} \; \Rightarrow_i \qquad\qquad \frac{\Gamma \vdash \phi \Rightarrow \psi \quad \Gamma \vdash \phi}{\Gamma \vdash \psi} \; \Rightarrow_e$$

$$\frac{\Gamma \vdash \phi}{\Gamma \vdash \phi \vee \psi} \; \vee_{i1} \quad \frac{\Gamma \vdash \psi}{\Gamma \vdash \phi \vee \psi} \; \vee_{i2} \qquad \frac{\Gamma \vdash \phi \vee \psi \quad \Gamma, \phi \vdash \theta \quad \Gamma, \psi \vdash \theta}{\Gamma \vdash \theta} \; \vee_{e1}$$

$$\frac{\Gamma, \neg\phi \vdash \perp}{\Gamma \vdash \phi} \; \perp_e$$

$$\frac{\Gamma, \phi \vdash \perp}{\Gamma \vdash \neg\phi} \; \neg_i \qquad\qquad \frac{\Gamma \vdash \phi \quad \Gamma \vdash \neg\phi}{\Gamma \vdash \psi} \; \neg_e$$

$$\frac{\Gamma \vdash \phi \quad x \; non \; libre \; dans \; \Gamma}{\Gamma \vdash \forall x \phi} \; \forall_i \qquad\qquad \frac{\Gamma \vdash \forall x \phi}{\Gamma \vdash \phi[x := t]} \; \forall_e$$

$$\frac{\Gamma \vdash \phi[x := t]}{\Gamma \vdash \exists x \phi} \; \exists_i \qquad \frac{\Gamma \vdash \exists x \phi \quad \Gamma, \phi \vdash \psi \quad x \; libre \; ni \; dans \; \Gamma ni \; dans \; \psi}{\Gamma \vdash \psi} \; \exists_e$$